Extended LP formulation for pump scheduling in water distribution networks

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drinking water distribution network



- nodes: reservoirs J_R , tanks J_T , junctions (demand nodes) J_J
- arcs: pipes L, pumps K, valves V

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- the historic day/night strategy is not compatible with dynamic tariffs
- a highly **combinatorial** $O(2^{K.T})$, highly **non-convex** scheduling problem

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pipe: head loss



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▶ good news: at time t and demand $D_t \in \mathbb{R}^{J_J}$, given a pump configuration $X \in \{0, 1\}^K$ and tank heads $H \in \mathbb{R}^{J_T}$, there is at most one possible flow/head $(q, h) \in \mathbb{R}^{L \times J}$ solution, which can quickly be computed with the Newton method (TODINI-PILATI88).

two main solution approaches

relax the NL part of MINLP

$\min \sum_{t \in T} \sum_{k \in K} C_t \Delta_t (C_k q_{kt} + E_k x_{kt})$	
$s.t. \sum_{ij\in L} q_{ijt} - \sum_{ji\in L} q_{jit} = D_{jt}$	$\forall t,j \in J_J$
$\sum_{ij \in L} q_{ijt} - \sum_{ji \in L} q_{jit} = \frac{S_j}{\Delta_t} (h_{jt} - h_{j(t-1)})$	$\forall t,j \in J_T$
$(h_{jt} - h_{it} + F_{ijt}q_{ijt}^2 + G_{ij})x_{ijt} = 0$	$\forall t, ij \in K$
$h_{it} - h_{jt} = A_{ij} q_{ijt} q_{ijt} + B_{ij}q_{ijt}$	$\forall t, ij \in L_p$
$H_j^{\min} \le h_{jt} \le H_j^{\max}$	$\forall t, j \in J_T$
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PWL approximation (MORSi12,MENKE16,...) convex relaxation (BONVIN17,BONVIN19)

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separate feasibility/optimization

choose configurations

 $\downarrow \uparrow$

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metaheuristics, ex: GA (MACKLE95,...), Benders decomposition (NAOUM15), lagrangian relaxation (GHADDAR15)

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many binaries

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slow convergence

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S. Demassey -- extended LP for pump scheduling

 $\forall t, j \in J_T$

 $\forall t, ij \in L_P$

 $\forall t, j \in J_T$

 $\forall t, k \in K$

the compact model

$$\begin{split} \min \ & \sum_{t \in T} \sum_{k \in K} C_t \Delta_t (C_k q_{kt} + E_k x_{kt}) \\ \text{s.t.} \ & \sum_{ij \in L} q_{ijt} - \sum_{ji \in L} q_{jit} = D_{jt} \\ & \sum_{ij \in L} q_{ijt} - \sum_{ji \in L} q_{jit} = \frac{S_j}{\Delta_t} (h_{jt} - h_{j(t-1)}) \\ & (h_{jt} - h_{it} + F_{ijt} q_{ijt}^2 + G_{ij}) x_{ijt} = 0 \\ & h_{it} - h_{jt} = A_{ij} |q_{ijt}| q_{ijt} + B_{ij} q_{ijt} \\ & H_j^{\min} \le h_{jt} \le H_j^{\max} \\ & q_{kt} \le Q_k^{\max} x_{kt} \\ & x_{kt} \in \{0, 1\} \end{split}$$

- min pump power consumption
- $\forall t, j \in J_J$ flow conservation at junctions
 - flow conservation at tanks
 - head increase by pumps
- $\forall t, ij \in K$ head losses in pipes
 - tank capacities
 - pump capacities
- $\forall t, k \in K$ > pumps on/off

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time coupling constraints

the compact model

rewrite one-time steps

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S_t set of feasible pump/flow/head configurations to supply demand D_t

compact model

extended model

$$\begin{split} \min \sum_{t \in T} \sum_{k \in K} C_t \Delta_t (C_k q_{kt} + E_k x_{kt}) & \min \sum_{t \in T} \sum_{s \in \mathcal{S}_t} C_t P^s \Delta_t y_{st} \\ s.t. \ h_{jt} - h_{j(t-1)} &= \frac{\Delta_t}{S_j} (\sum_{ij \in L} q_{ijt} - \sum_{ji \in L} q_{jit}) \quad \forall t, j \in J_T \\ (x_t, q_t, h_t) \in \mathcal{S}_t & \forall t & h_{jt} = \sum_{s \in \mathcal{S}_t} H_j^s y_{st} & \forall t, j \in J_T \\ & \sum_{s \in \mathcal{S}_t} y_{st} = 1 & \forall t \\ & y_{st} \in \{0, 1\} & \forall t, s \in \mathcal{S}_t \end{split}$$

- $P \in \mathbb{R}$ power consumption, $R \in \mathbb{R}^{J_T}$ tank filling rate, $H \in \mathbb{R}^{J_T}$ tank head
- ► $|S_t| = \infty$ but from (Todini&Pilati88):
- ▶ (at most) one $s \in S_t$ for each $x_t \in \{0, 1\}^K$ and $h_t \in [H_t^{\min}, H_t^{\max}] \subseteq \mathbb{R}^{J_T}$

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proposed approximation

extended model

relax the integrality constraints

$$\begin{split} \min \sum_{t \in T} \sum_{s \in \mathcal{S}_t} C_t P^s \Delta_t y_{st} \\ \text{s.t. } h_{jt} - h_{j(t-1)} &= \sum_{s \in \mathcal{S}_t} R_j^s \Delta_t y_{st} \quad \forall t, j \in J_T \\ h_{jt} &= \sum_{s \in \mathcal{S}_t} H_j^s y_{st} \qquad \forall t, j \in J_T \\ \sum_{s \in \mathcal{S}_t} y_{st} &= 1 \qquad \forall t \\ y_{st} &\geq 0 \qquad \forall t, s \in \mathcal{S}_t \end{split}$$

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- relax the head/configuration linking constraint

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- relax the integrality constraints
- relax the head/configuration linking constraint
- ► restrict to columns $s \in S'_t \subseteq S_t$ with $H^s_j = \frac{H^{\max}_j - H^{\min}_j}{2}$, $\forall j \in J_T$

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▶ $|S'_t| < 2^{|K|}$ AND can be computed efficiently

▶ apply Newton method: fix $D_t \in \mathbb{R}^{J_J}$ and $H_t \in \mathbb{R}^{J_T}$, then compute Q^s then (P^s, R^s) (if feasible) for all $X^s \in \{0, 1\}^K$

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choose the sampling step carefully

$N_k + 1$	2	3	4	5	6	7	8	9	10
$ \mathcal{S}'_t $	21	52	105	186	301	456	657	910	1221
Z'	244.44	242.15	215.62	215.34	217.50	213.95	211.70	212.97	212.20

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▶ also for pressure-reducing valves: either open or pressure reduction $p_v \in [p_v^{\min}, p_v^{\max}]$

approximated solution

solve the extended LP model and get for each time t the active configurations C^{*}_t = {s ∈ S'_t | y_{st} > 0} of durations δ^{*}_s = Δ_ty_{st}.
 Order each set C^{*}_t arbitrarily and get an approximated pumping plan:

$$P^* = \underbrace{s_0, s_1, \dots, s_{n_0}}_{C_0^*}, \underbrace{s_{n_0+1}, \dots, s_{n_0+n_1}}_{C_1^*}, \dots, \underbrace{\ldots}_{C_{T-1}^*}$$

start with i = 0, apply the Newton method to s_i with $H_i \in \mathbb{R}^{J_T}$ to get the actual flow rates Q_i , then compute the filling rates R_i and update tank heads $H_{i+1} = H_i + \delta_i R_i$.

▶ plan
$$P^*$$
 is valid if $H^{\min} \leq H_i \leq H^{\max}$ for all *i*

to a close feasible solution

- each pump can be switched at any time... not in any old way
- operational constraints to prevent premature aging, e.g N max nb of switches on, τ_0/τ_1 max nb of consecutive times off/on

$$\begin{split} &\sum_{t \in \mathcal{T}} y_{kt} \leq N, \\ &y_{kt} \geq x_{kt} - x_{k(t-1)}, & \forall t \\ &x_{kt'} \geq y_{kt}, & \forall t, t' \in [t, t + \tau_1] \\ &z_{kt} \geq x_{k(t-1)} - x_{kt}, & \forall t \\ &x_{kt'} \leq 1 - z_{kt}, & \forall t, t' \in [t, t + \tau_0] \end{split}$$

Find a **feasible** plan P (with one configuration per time step, satisfying tank capacities and operational constraints) at a close distance of P^* i.e. with $\delta_{kt} \approx \delta^*_{kt}$, the activity duration of pump k in time step t

combinatorial Benders local search

- ► solve (M) : min $\sum_{k} \sum_{t} (\delta_{kt}^* x_{kt}\Delta_t)^2 + \sum_{k \in K} (\sum_{t} \delta_{kt}^* \sum_{t} x_{kt}\Delta_t)^2$ s.t.: (operational constraints), $x \in \{0, 1\}^{K \times T}$
- apply Newton method iteratively on each configuration x_t , t = 0, ..., T 1, and get the actual flows-heads (Q, H)
- If some constraint is violated at time t, add to (M) a no-good constraint

$$\sum_{t=1}^{\tilde{t}} \left(\sum_{\substack{k \in K \\ X_{kt}=0}} x_{kt} + \sum_{\substack{k \in K \\ X_{kt}=1}} (1 - x_{kt}) \right) \ge 1$$

• try to correct the small violations by adjusting the time step durations Δ_t using the matheuristic from (BONVIN-DEMASSEY-LODI 19)

near-feasible approximated solutions P^*



Poormond instance (GHADDAR15)





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- $\blacktriangleright Z = 111.03, Z^* = 117.5$ euros





- best and LB computed in 1h with LP/NLP branch and check (BONVIN19)
- Van Zyl (sampling 6 speeds/3 pumps, 1 valve, |T| = 48)
 - $6^3 \times 2 \times 48 \approx 20,000$ configurations to evaluate
 - network decomposition: $6^3 \times 2 + 48 = 480$ to compute
 - P*: 50 active configurations
 - get P by solving the compact NLP with fixed X

limits and perspectives

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- still an exponential number of configurations to compute: could we build S'_t from historical data ?
- no optimality certificate: how to integrate the approximated model into a global optimization approach ?