ENHANCED BRANCH & CHECK FOR PUMP SCHEDULING IN WATER NETWORKS

Sophie Demassey (Mines ParisTech / PSL)
elaborating upon a joint work with Gratien Bonvin and Andrea Lodi
MINLP 2021 Workshop

WATER MOTION

- water falls (from high to low potentials)

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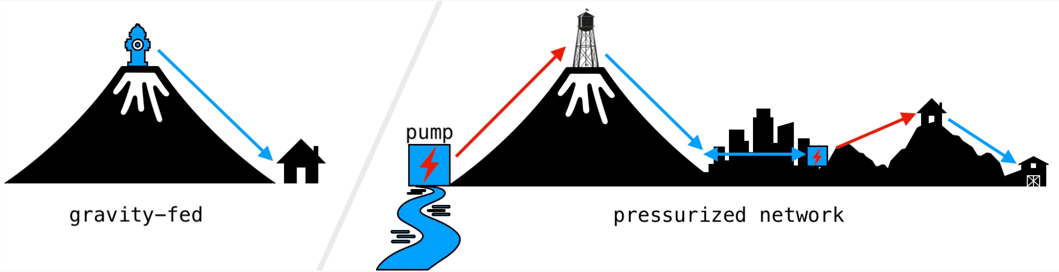

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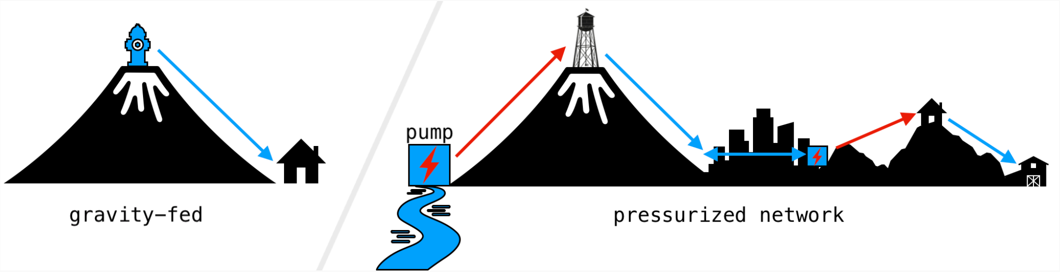
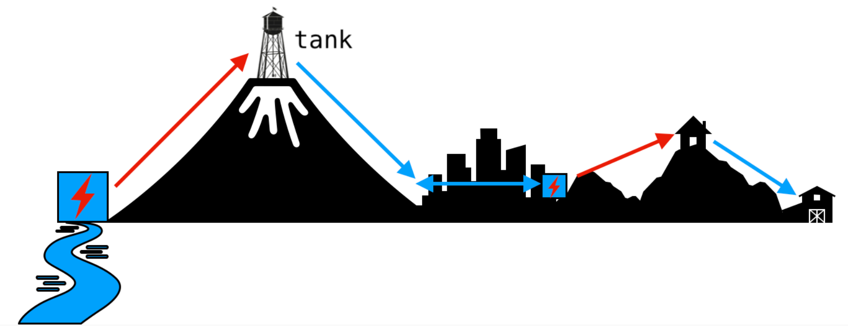


- pressurized networks are more common today
- they consume electricity

ENERGY-INTENSIVE BUT FLEXIBLE



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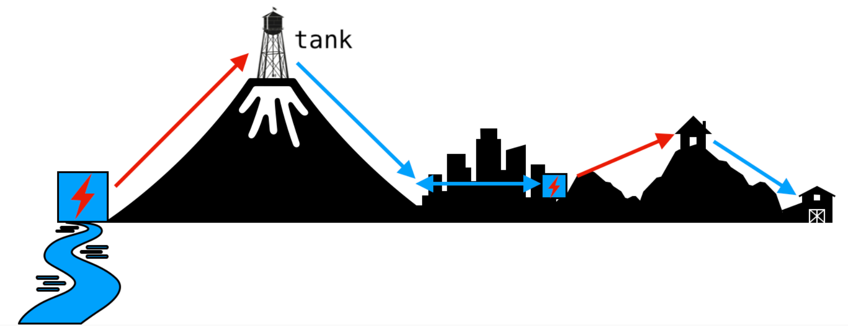

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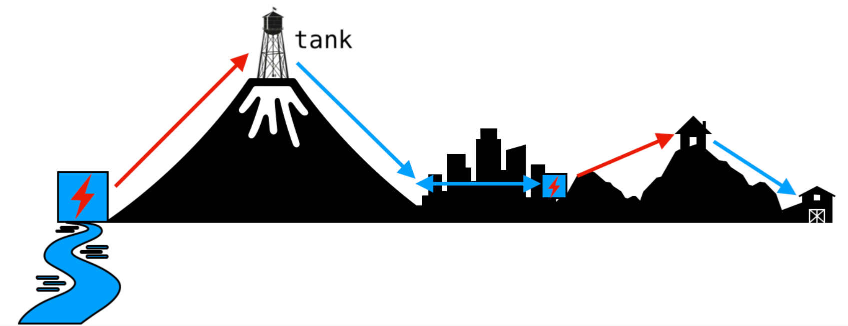

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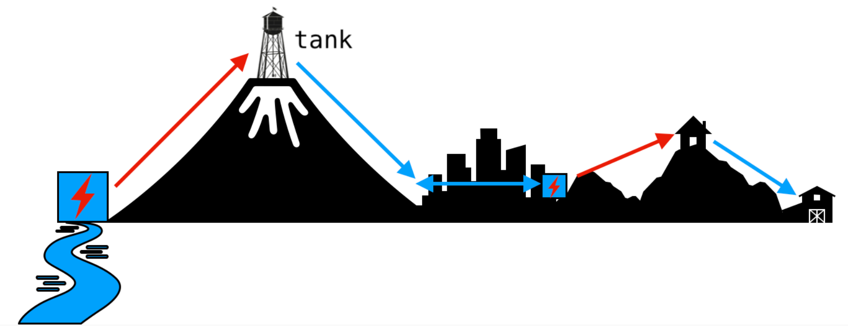
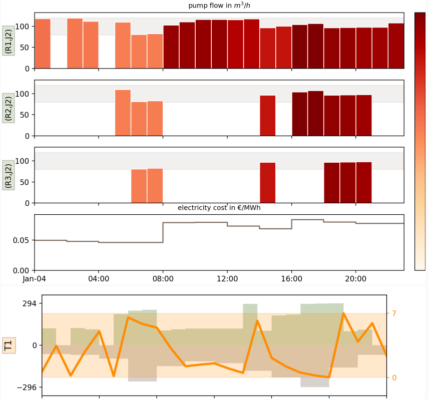


- reduce the electricity bill
- using elevated tanks as buffers to shift pumping
- when electricity is cheaper/for better efficiency
- tanks have limited capacities

PUMP SCHEDULING

when/how to pump over a discretized horizon $t = 1, \dots, \mathcal{T}$ to:

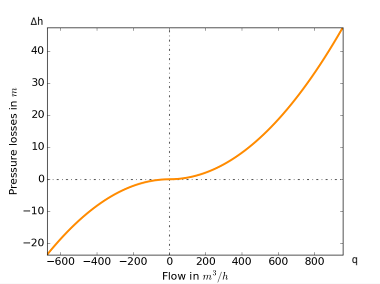
- minimize the electricity cost
- meet the forecast demand profiles
- respect the tank capacities
- satisfy the steady-state flow/potential relations at each t



NONCONVEXITY (NO VSD PUMPS/PR VALVES)

potential-flow relation on each arc=pipe/pump (i, j) and time t

$$h_{it} - h_{jt} = \phi_{ij}(q_{ijt})$$



- flow sign = flow direction
- accurate approximation as an antisymetric quadratic function:

$$\phi(q) = \alpha|q|q + \beta q + \kappa \quad \text{with} \quad \alpha > 0$$

- friction in pipes: $\kappa = 0$
- active (fixed-speed) pumps: $\kappa < 0$ and $q > 0$

LITERATURE

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- [Burgschweiger 2009]: *Optimization models for operative planning in drinking water networks*
- [Ghaddar 2015]: *A lagrangian decomposition approach for the pump scheduling problem in water networks*
- [Naoum-Sawaya 2015]: *Simulation-optimization approaches for water pump scheduling and pipe replacement problems*
- [Shi&You 2016]: *Energy optimization of water supply system scheduling: Novel MINLP model and efficient global optimization algorithm*

- [Bonvin 2018] *Contrôle optimal et dimensionnement des stations de pompage dans les réseaux de distribution d'eau potable*
- [Bonvin et al. 2017] *A convex mathematical program for pump scheduling in a class of branched water networks*
- [Bonvin, Demassey, Lodi 2021] *Pump scheduling in drinking water distribution networks with an LP/NLP-based branch and bound*
- [Bonvin, Demassey 2019] *Extended linear formulation of the pump scheduling problem in water distribution networks*
- [Bonvin, Demassey, de Oliveira 2019] *Robust design of pumping stations in water networks*

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BRING ON THE MATHS

1. implementation of branch&check for nonconvex MINLP over a MILP solver with user callbacks and lazy cuts
2. managing the incumbent
3. strong duality cuts

BRANCH&CHECK FOR NONCONVEX MINLP

$$MINLP : \min_{x,y}\{f(x,y) \mid g(x,y) \leq 0, x \in \mathbb{B}^n, y \in \mathbb{R}^p\}$$

- get a MILP relaxation $OA : \min_{x,y}\{f(x,y) \mid \bar{g}(x,y) \leq 0, x \in \mathbb{B}^n, y \in \mathbb{R}^p\}$
- solve with a branch&cut where

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- solve with a branch&cut where
- at each integer solution $x = X$, solve $NLP(X) : \min_{y}\{f(X,y) \mid g(X,y) \leq 0, y \in \mathbb{R}^p\}$
    - if unfeasible: add a nogood cut $\|x - X\|_1 \geq 1$ to OA
    - if feasible and improving: update the incumbent $UB = \min(UB, f(X, Y^X))$ and cut off $f(x,y) \leq UB - \epsilon$
    - if feasible: discard the relaxed solution $(X, \bar{Y})$

IMPLEMENTATION IN MILP SOLVERS

- user callbacks: interrupt the solver at chosen (integer) nodes
- lazy constraints: add nogood, cutoff or OA cuts (on convex side)
- local constraints: OA cuts with local tighter bounds (not supported in Gurobi 9)
- repair unfeasible solutions heuristically
- detect smaller conflicts $X' \subseteq X$ to get stronger nogood cuts

MANAGING THE INCUMBENT

The solver underestimates the value at feasible nodes $x = X$:
$LB = f(X, \bar{Y}) \leq f(X, Y^X)$

Different solver-dependent answers:

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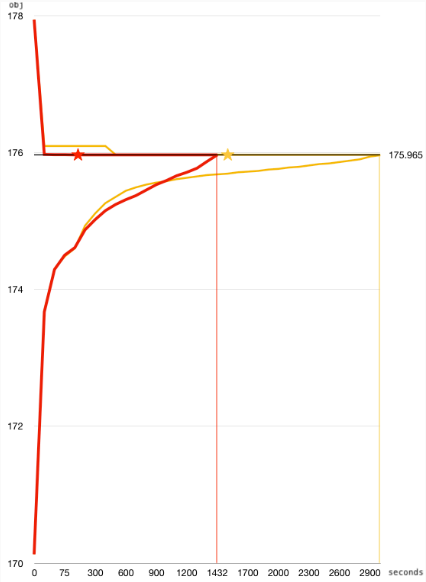
   - no such "lazy best solution" functionality in Gurobi 9
3. workaround: provide the real solution to the solver and cut the relaxed solution with:

$$f(x, y) \geq f(X, Y^X) * (1 - \|x - X\|_1)$$

MANAGING THE INCUMBENT

- bounding vs. fathoming
  evolution of the primal/upper and dual/lower bounds
- simple network / T=48
- **non-contractual** (but representative): single instance, single run



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- derive the strong duality condition $F(X,y) \leq L(X,u)$
- linearize the strong duality constraint $F(x,y) - L(x,u) \leq 0$

NONLINEAR NETWORK ANALYSIS PROBLEM

steady-state potential/flow equilibrium in a digraph $G = (N, A)$ where nodes $N = J \cup R$ have either a known demand d_J or a known potential h_R

network analysis problem

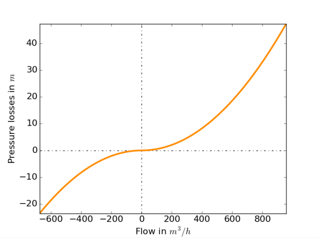
$$
\begin{aligned}
NL(A, d_J, h_R) = \{(q_A, h_J) \in \mathbb{R}^A \times \mathbb{R}^J \\
q_j = d_j \qquad & \forall j \in J, \\
h_a = \phi_a(q_a) \qquad & \forall a \in A\}.
\end{aligned}
$$

$h_{(i,j)} = h_i - h_j$: potential decrease along arc $(i, j) \in A$

$q_j = \sum_{(i,j)\in A} q_{(i,j)} - \sum_{(j,i)\in A} q_{(j,i)}$: residual flow at node $j \in N$,

ASSUMPTIONS

- ϕ_a bijective, smooth, strictly increasing on $\mathbb{R}$ so that
  - ϕ_a invertible
  - $\Phi_a(q) = \int_0^q \phi(s)ds$ coercive, smooth, strictly convex.
- at least one fixed potential node $j \in R$ in each connected component of G so that:
  - $q_J = d_J$ is feasible
  - h_J is uniquely determined by q_A



Lagrangian multiplier theorem holds:

$(q_A, h_J) \in NL(A, d_J, h_R)$ for some h_J if and only if q_A solves

$$P(A, q_J, h_R) : \min_{q_A}\{f(q_A) = \sum_{a\in A} \Phi_a(q_a) + h_R q_R, q_J = d_J\}$$

- by convexity of Φ_a (KKT necessary+sufficient opt. cond.)

$$L(q_A, h_J) = \sum_{a\in A} \Phi_a(q_a) + h_R q_R + h_J(q_J - d_J)$$

$$\begin{cases} h_A = \phi_A(q_A) & \text{(1st-order cond. } \frac{\partial L}{\partial q_A} = 0), \\ q_J = d_J & \text{(primal feas. } \frac{\partial L}{\partial h_A} = 0) \end{cases}$$

- solution exists and is unique (by assumptions on G and strict convexity of f)

CONVEX OPTIMIZATION: DUAL

- by convexity of f and unicity of h_J:

Strong duality holds:

$(q_A, h_J) \in NL(A, d_J, h_R)$ if and only if
$q_J = d_J$ and $f(q_A) \leq L(h_J) = \min_{q_A} L(q_A, h_J)$.

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$q_J = d_J$ and $f(q_A) \leq L(h_J) = \min_{q_A} L(q_A, h_J)$.

- $L(q_A, h_J) = \sum_{a \in A}(\Phi_a(q_a) - h_a q_a) - h_J d_J$ and $q_a \mapsto \Phi_a(q_a) - h_a q_a$ is convex and reaches its minimum at $q_a = \phi_a^{-1}(h_a)$ then:

Decomposition of the dual function:

$$L(h_J) = \sum_A L_a(h_a) - h_J d_J$$

with $L_a(h_a) = \Phi_a(\phi_a^{-1}(h_a)) - h_a \phi_a^{-1}(h_a)$.

CONVEX NLP REFORMULATION

$NL(A, d_J, h_R) = NL'(A, d_J, h_R)$ with

$$NL'(A, q_J, h_R) = \{\, (q_A, h_J) \in \mathbb{R}^A \times \mathbb{R}^J$$

$$\sum_{a \in A} g_a(q_a, h_a) + h_N q_N \leq 0$$

$$q_J = d_J \,\}.$$

with $g_a(q_a, h_a) = \Phi_a(q_a) - L_a(h_a)$ convex (by assumption on ϕ_a).

ANOTHER VIEW OF PUMP SCHEDULING

minimize some cost function over the solutions of $\mathcal{T}$ sequence-dependent nonlinear analysis problems on a dynamic network:

- controllable arcs $x_{at} \in \{0, 1\}$: pumps and valved pipes
- nodes with varying known demand d_{jt}: junctions
- nodes with varying known potential h_{rt}: sources ($r \in S \subseteq R$)
- nodes with bounded sequence-dependent potential h_{rt}: tanks ($r \in T \subseteq R$)

$$
\begin{aligned}
h_{r0} &= H_r \\
h_{r(t+1)} &= h_{rt} + \alpha_r q_{rt} \\
\underline{H}_{rt} &\leq h_{rt} \leq \overline{H}_{rt}.
\end{aligned}
$$

THE RESTRICTED SUBPROBLEM

For a fixed $x = X \in \{0,1\}^{A\times\mathcal{T}}$, $(q_{A\mathcal{T}}, h_{N\mathcal{T}})$ solves $NLP(X_{A\mathcal{T}})$ iff

$$
\begin{aligned}
&(q_{At}, h_{Jt}) \in NL(AX_t, d_{Jt}, h_{Rt}) && \forall\, t \\
&X_{at} = 0 \implies q_{at} = 0 && \forall\, t, a \\
&h_{T(t+1)} = h_{Tt} + \alpha_T q_{Tt} && \forall\, t \\
&h_{T0)} = H_T, h_{T(\mathcal{T}+1)} \geq H_T && \\
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strong duality constraint

$$\sum_{a \in A} g_a(q_{at}, h_{at}) x_{at} + h_{Nt} q_{Nt} \leq 0$$

is valid at each period t.

STRONG DUALITY CONSTRAINT

The strong duality constraint

$$\sum_{a \in A} g_a(q_{at}, h_{at}) x_{at} + h_{Tt} q_{Tt} + h_{St} q_{St} + h_{Jt} d_{Jt} \leq 0$$

- non-convex because of the bilinear terms $h_{rt} q_{rt}$, $r \in T$
- the bad news: the strong duality constraint is an aggregated form of $\{h_{at} = \phi_a(q_{at}), a \in A\}$ and a loose relaxation of $h_{rt} q_{rt}$ could "*absorb*" the duality gap
- the good news: $|T| << |A|$ and the tank capacities provide constrained bounds: $h_{rt} \in [\underline{H}_r, \overline{H}_r]$ and $q_{rt} = (h_{r(t+1)} - h_{rt})/\alpha_r$

The strong duality constraint

$$\sum_{a \in A} g_a(q_{at}, h_{at}) x_{at} + h_{Tt} q_{Tt} + h_{St} q_{St} + h_{Jt} d_{Jt} \leq 0$$

Linearize g_a at some feasible points $(q_a^*, \phi_a(q_a)^*)$ and take the McCormick's envelope for the bilinear terms $h_{rt} q_{rt}$, $r \in T$:

$$\sum_{a \in A} g_{at} + h'_{Rt} + h_{St} q_{St} + h_{Jt} d_{Jt} \leq 0$$

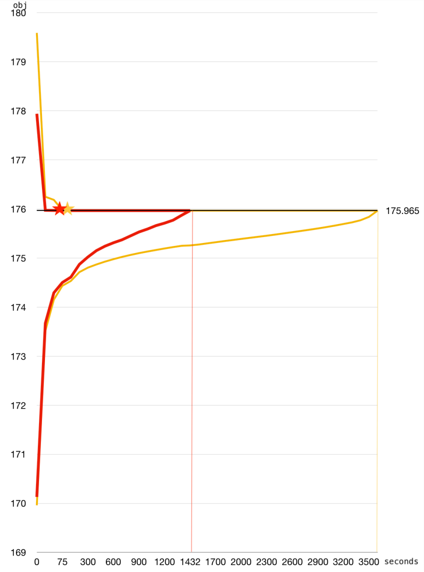
$$\begin{aligned}
x_{at} &= 0 \implies q_{at} = 0 \wedge h_{at} = 0 && \forall a \\
x_{at} &= 1 \implies h_{at} = h_{it} - h_{jt} && \forall a = (i, j) \\
g_{at} &\geq \phi_a(q_a^*)(q_{at} - q_a^* x_{at}) + q_a^* h_{at} && \forall a, q_a^* \in \mathcal{Q}_a \subseteq [\underline{Q}_a, \overline{Q}_a] \\
h'_{rt} &\in MC_{[\underline{H}_r, \overline{H}_r]}(h_{rt} q_{rt}) && \forall r \in T.
\end{aligned}$$

STRONG DUALITY CONSTRAINTS

with or without duality constraints in initial OA
5 linear/pipes, 10/pumps

primal/dual bounds
simple network / T=48

non-contractual (but representative)



RELATED WORKS FOR THE OPTIMAL PIPE SIZING PROBLEM

aka: gravity-fed water network design problem

- [Raghunatan 2013]: *Global optimization of nonlinear network design*
  - branch&check + OA cut generation
  - without the incumbent management issue (the objective $f(x)$ only depends on the binary variables)
  - with several problem-specific improvements: repair heuristic, aggregated linearizations

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  - with several problem-specific improvements: repair heuristic, aggregated linearizations
- [TASSEF, BENT, EPELMAN, PASQUALINI, VAN HENTENRYCK (ARXIV 2020)]: *Exact Mixed-integer Convex Programming Formulation for Optimal Water Network Design*
  - the strong duality constraint is directly convex (no tank, no bilinear term)
  - nice physical interpretation of the constraint

Other possible applications: operation/design/planning of crude oil/natural gas/electricity/transportation networks ?

REFERENCES

- papers by Bonvin et al. available on `https://sofdem.github.io/`
- code (partially) available on: `https://github.com/sofdem/gopslpnlpbb`