Alternating direction methods for scheduling with storage

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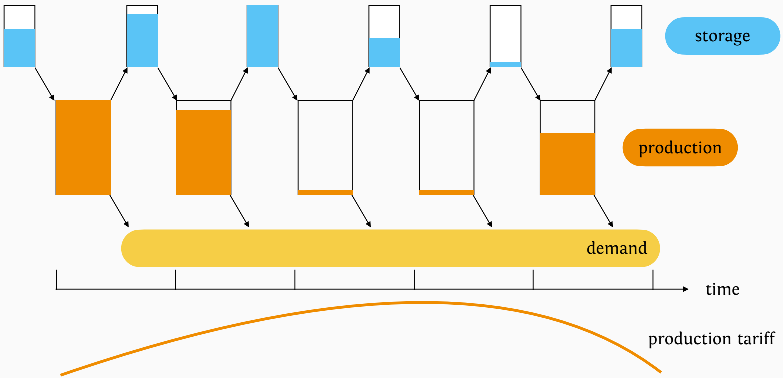
SCHEDULING WITH STORAGE



decide on operation and storage levels
to meet demand, capacity, and flow conservation on all periods
and minimize the total operation cost (**load shifting**)

SCHEDULING WITH STORAGE

$$(P) : \min_{x,y,s} \sum_{t \in \mathcal{T}} f_t(x_t, y_t, s_t, C_t) \tag{1}$$

$$s.t. : g_t(x_t, y_t, s_t, L_t) = 0 \qquad \forall\, t \in \mathcal{T} \tag{2}$$

$$s_{t+1} = s_t + y_t^I \qquad \forall\, t \in \mathcal{T} \tag{3}$$

$$s_t \in \mathcal{S}_t = [\underline{S}_t, \overline{S}_t] \subseteq \mathbb{R}^I \qquad \forall\, t \in \overline{\mathcal{T}} \tag{4}$$

$$x_t \in \mathcal{X}_t \subseteq \{0,1\}^N, y_t \in \mathcal{Y}_t \subseteq \mathbb{R}^M \qquad \forall\, t \in \mathcal{T}. \tag{5}$$

decide on operation (x_t, y_t) and storage s_t levels
to meet demand (2), capacity (4) and flow conservation (3) on all periods t
and minimize the total operation cost (1)

ASSUMPTION ON THE STEADY STATE

steady state operation (x_t, y_t) for given storage level s_t and demand L_t

$$(x_t, y_t) \;:\; g_t(x_t, y_t, s_t, L_t) = 0$$

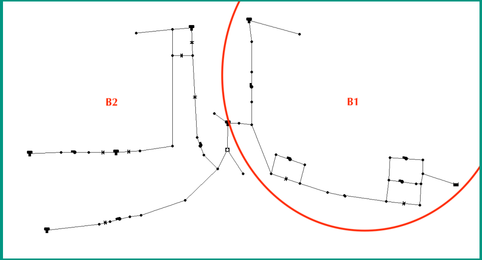
a possibly nonconvex system, but assume that it is easy to solve and optimize on if s_t **is fixed**

EX 1: SCHEDULING OF POTENTIAL-FLOW NETWORKS

sequence of potential-flow equilibria on a dynamic graph

$$x_t^\top (y_t^H - \phi(y_t^Q)) = 0,\ y_{tJ}^Q = L_t,\ y_{tR}^H = s_t$$

x_t: on/off activity of the arcs,
(y_t^Q, y_t^H): active arc flows, nodal potentials
s_t: potential at storage nodes,
L_t: demand at service nodes



- nonconvex system (potential-flow relation ϕ_a on each arc)
- for x_t and s_t fixed: (y_t^Q, y_t^H) unique KKT solution of a linearly-constrained strictly convex problem
- for s_t fixed: $\min_{x_t \in \{0,1\}^A} f_t(x_t)$ is enumerable with graph partition along the tanks

EX 2: EXPANSION PLANNING W/WO STORAGE

fine-grained schedule on a coarse-grained period t

$$g_t^i(x_t^i, y_t^i, s_t^i, L_t^i) = 0 \;\forall\, i \in \mathbb{I}_t,\; s_t^{i+1} = s_t^i + y_t^i \;\forall\, i \in \mathbb{I}_t,\; s_t^0 = s_t$$

(x_t, y_t): fine-grained operation+investment on the period

s_t: available capacity/storage at the beginning of the period

L_t: fine-grained demand on the period

- each subproblem is easy to optimize for s_t fixed, as the horizon is smaller
- optimizing with s_t variable may lead to all-or-none solutions, e.g.: $s_t = \overline{S}_t$ and $y_t = 0$.

OPTION 0: DUALIZE THE TIME-COUPLING CONSTRAINTS

ex: lagrangian subproblem

$$(P): \min_{x,y,s} \sum_{t\in\mathcal{T}} f_t(x_t, y_t, s_t, C_t) + \mu_t(s_{t+1} - s_t - y_t^I)$$

$$s.t.: g_t(x_t, y_t, s_t, L_t) = 0 \qquad \forall t \in \mathcal{T}$$

- the model becomes separable in time

$$\sum_{t\in\mathcal{T}} \min_{x_t,y_t,s_t} \{f_t(x_t, y_t, s_t) + (\mu_t - \mu_{t-1})^\top s_t + \mu_t^\top y_t : g_t(x_t, y_t, s_t, L_t) = 0\}.$$

- not separable with penalty terms, e.g. quadratic $\frac{\rho_t}{2}|s_{t+1} - s_t - y_t|^2$
- s_t **is variable** so each subproblem remains **hard** (potential/flow) or **poor** (hierarchical planning)

OPTION 1: FULL VARIABLE-SPLIT AND ADMM

ADMM: variant of the augmented lagrangian $p_t(z,\rho) = \rho_d^\top z + \rho_p||z||_2$ with partial update

1: fix storage s, then compute (x,y)

$$P(s):\min_{(x,y)\in\mathcal{X}\times\mathcal{Y}}\sum_{t\in\mathcal{T}} f_t(x_t,y_t,s_t,C_t)+p_t(s_{t+1}-s_t-y_t,\rho_t)+p_t(g_t(x_t,y_t,s_t,L_t),\mu_t).$$

$\downarrow \quad \uparrow \quad$ update ρ, μ

2: fix command (x,y), then compute s

$$P(x,y):\min_{s\in\mathcal{S}}\sum_{t\in\mathcal{T}} f_t(x_t,y_t,s_t,C_t)+p_t(s_{t+1}-(s_t+y_t),\rho_t)+p_t(g_t(x_t,y_t,s_t,L_t),\mu_t)$$

$$s.t.: s_{t+1}=s_t+y_t^I \qquad \forall t\in\mathcal{T}.$$

- strong theoretical convergence, even with nonconvexity (ex: OPF) **not in the coupling constraints**
- $P(s)$ is too poor, $P(x,y)$ too hard

OPTION 2: PARTIAL SPLIT AND ADM-LIKE

if no theoretical convergence result exists, let's make it practical

1: fix storage s, then compute (x, y)

$$P(s): \min_{(x,y)\in\mathcal{X}\times\mathcal{Y}} \sum_{t\in\mathcal{T}} f_t(x_t, y_t, s_t, C_t) + p_t(s_{t+1} - s_t - y_t, \rho_t)$$

$$s.t.: g_t(x_t, y_t, s_t, L_t) = 0 \qquad \forall t \in \mathcal{T}$$

↓ ↑ stop when $\|s_{t+1} - s_t - y_t\| < \epsilon$

2: fix command (x, y), then compute s

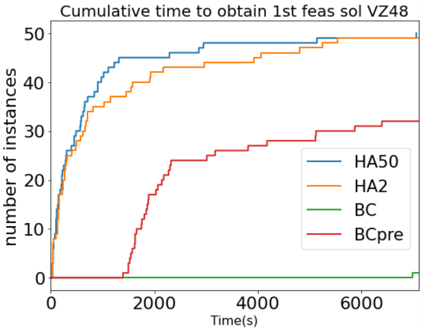
$$P(x, y): \min_{s\in\mathcal{S}} \sum_{t\in\mathcal{T}} f_t(x_t, y_t, s_t, C_t) + p_t(s_{t+1} - s_t - y_t, \rho_t)$$

$$s.t.: s_{t+1} = s_t + y_t^I \qquad \forall t \in \mathcal{T}.$$

keep $g_t(x_t, y_t, s_t, L_t) = 0$ in $P(s)$ (easy), but drop it from $P(x, y)$ (inverse problem)

Experiments: pump scheduling in water networks

- **HA**: partial split $\rho_0 \in \{50, 2\}$ + initial storage profiles learned with DL [ISCO 2024]
- **BC**: SOA Branch-and-Check [Opt&Eng 2021] + **BCpre** advanced preprocessing [ICAE 2022]
- run algorithms on 50 instances within 2 hours; stop at the first feasible solution
- hard to just compute a feasible solution when storage limits are tight



Reconsider full-split with bilevel model

Ex: static potential/flow equilibrium, given x and s (and time t), let

$$Y(x,s) = \{y = (y^Q, y^H) : g(x,y,s,L) = 0\}$$

- $Y(x,s)$ are the **KKT solutions** for a min-strictly-convex-cost flow problem on $G(N, A(x))$:

$$
\begin{aligned}
& y^Q \in \arg\min_q \{\Phi(q) + s^\top q_R : q_J = L\} && \text{(primal flow)} \\
\equiv\quad & y^H \in \arg\max_h \{\Phi^*(h) + L^\top h : h_R = s\} && \text{(dual potential)} \\
\equiv\quad & \Phi(y^Q) + s^\top y^Q_R = \Phi^*(y^H) + L^\top y^H,\ y^Q_J = L,\ y^H_R = s && \text{(strong duality condition).}
\end{aligned}
$$

- with a suited variable change, x does not appear in the SD condition
- nonconvexity remains in the bilinear term $s^\top y^Q_R$

DUALIZE THE SD CONDITION + ADMM

1: fix storage s, then compute (x, y)

$$P(s): \min_{(x,y)\in\mathcal{X}\times\mathcal{Y}} \sum_{t\in\mathcal{T}} f_t(x_t, y_t, s_t, C_t) + p_t(s_{t+1} - s_t - y_t, \rho_t) + p_t(SD_t(y_t, s_t), \mu_t)$$

$$s.t.: y^Q_{tJ} = L,\ (1 - x_t)y^Q_t = 0,\ y^H_{tR} = s_t,\ y^H_t \in B(x_t) \qquad \forall t \in \mathcal{T}.$$

with $SD_t(y_t, s_t) = \Phi(y^Q_t) + s_t^\top y^Q_{tR} - \Phi^*(y^H_t) - L_t^\top y^H_t$ (and f_t, p_t linear),

DUALIZE THE SD CONDITION + ADMM

1: fix storage s, then compute (x, y)

$$P(s): \min_{(x,y)\in\mathcal{X}\times\mathcal{Y}} \sum_{t\in\mathcal{T}} f_t(x_t, y_t, s_t, C_t) + p_t(s_{t+1} - s_t - y_t, \rho_t) + p_t(SD_t(y_t, s_t), \mu_t)$$

$$s.t.: y_{tJ}^Q = L,\ (1 - x_t)y_t^Q = 0,\ y_{tR}^H = s_t,\ y_t^H \in B(x_t) \qquad \forall t \in \mathcal{T}.$$

with $SD_t(y_t, s_t) = \Phi(y_t^Q) + s_t^\top y_{tR}^Q - \Phi^*(y_t^H) - L_t^\top y_t^H$ (and f_t, p_t linear), then for each t, $P_t(s_t)$ is **separable in primal/dual parts**, i.e. (y^Q, y^H)-split, corresponding to two equilibrium problems perturbed with costs and penalties

primal: perturbed potentials s_t and resistance ϕ

$$P_t(x_t, s_t): \min_{y_t^Q} \mu_t \Phi(y_t^Q) + l(s_t, C_t, \rho_t, \mu_t)^\top y_t^Q$$

$$s.t.: y_{tJ}^Q = L_t,\ (1 - x_t)y_t^Q = 0.$$

dual: perturbed load L_t and resistance ϕ

$$D_t(x_t, s_t): \max_{y_t^H} \mu_t \Phi^*(y_t^H) + \mu_t L^\top y_t^H$$

$$s.t.: y_{tJ}^H = s_t,\ y_t^H \in B(x_t).$$

Conclusion

- coupling constraints ? consider regularization + alternating direction methods
- cascading separation: time $\longrightarrow$ space $\longrightarrow$ primal/dual
- initialization point (here the storage profiles) can be learned
- future: other applications (traffic, MPEC) and theoretical convergence

References

- ADMM: cf **Boyd** on proximal algorithms, apps in ML and OPF
- potential networks: cf **Rockafellar** on nonlinear flows and monotropic programming

- [ISCO 2024] **S.D., V. Sessa, A. Tavakoli** Deep learning and alternating direction method for discrete control with storage. In International Symposium on Combinatorial Optimization 2024.
- [ICAE 2022] **A. Tavakoli, V. Sessa, S.D.** Strengthening mathematical models for pump scheduling in water distribution. In 14th International Conference on Applied Energy 2022.
- [Opt&Eng 2021] **G. Bonvin, S.D., A. Lodi** Pump scheduling in drinking water distribution networks with an LP/NLP-based branch and bound. Optimization and Engineering 2021.

- papers available at `https://sofdem.github.io/`
- code available at `https://github.com/sofdem/gopslpnlpbb`