5.4 all_differ_from_exactly_k_pos

<p style="text-align:center">DESCRIPTION LINKS GRAPH</p>

Origin Inspired by all_differ_from_at_least_k_pos.

Constraint all_differ_from_exactly_k_pos(K, VECTORS)

Type VECTOR : collection(var−dvar)

Arguments
K : int
VECTORS : collection(vec − VECTOR)

Restrictions
required(VECTOR, var)
|VECTOR| ≥ 1
|VECTOR| ≥ K
K ≥ 0
required(VECTORS, vec)
same_size(VECTORS, vec)

Purpose Enforce all pairs of distinct vectors of the VECTORS collection to differ from exactly K positions. Enforce K = 0 when |VECTORS| < 2.

Example $(2, \langle \text{vec} − \langle 0,3,0,6 \rangle, \text{vec} − \langle 0,3,4,1 \rangle, \text{vec} − \langle 9,3,4,6 \rangle \rangle)$

The all_differ_from_exactly_k_pos constraint holds since:

- The first and second vectors differ from 2 positions, which is equal to K = 2.
- The first and third vectors differ from 2 positions, which is equal to K = 2.
- The second and third vectors differ from 2 positions, which is equal to K = 2.

Typical
K > 0
K < |VECTOR|
|VECTORS| > 1

Symmetries
- Items of VECTORS are permutable.
- Items of VECTORS.vec are permutable (*same permutation used*).

Arg. properties Contractible wrt. VECTORS.

See also **implies:** all_differ_from_at_least_k_pos (= K *replaced by* ≥ K), all_differ_from_at_most_k_pos (= K *replaced by* ≤ K).

part of system of constraints: differ_from_exactly_k_pos.

used in graph description: differ_from_exactly_k_pos.

Keywords   **characteristic of a constraint:** disequality, vector.

                     constraint type: system of constraints, decomposition.

                     final graph structure: no loop, symmetric.

Cond. implications   all_differ_from_exactly_k_pos(K, VECTORS)
                            with K ≤ |VECTORS|
                           implies atleast_nvector(NVEC, VECTORS).

Arc input(s)	VECTORS
Arc generator	$CLIQUE(\neq) \mapsto$ collection(vectors1, vectors2)
Arc arity	2
Arc constraint(s)	differ_from_exactly_k_pos(K, vectors1.vec, vectors2.vec)
Graph property(ies)	**NARC**= |VECTORS| $*$ |VECTORS| $-$ |VECTORS|
Graph class	• NO_LOOP • SYMMETRIC

Graph model

The **Arc constraint(s)** slot uses the differ_from_exactly_k_pos constraint defined in this catalogue.

Parts (A) and (B) of Figure 5.4 respectively show the initial and final graph associated with the **Example** slot. Since we use the **NARC** graph property, the arcs of the final graph are stressed in bold. The previous constraint holds since exactly $3 \cdot (3-1) = 6$ arc constraints hold.
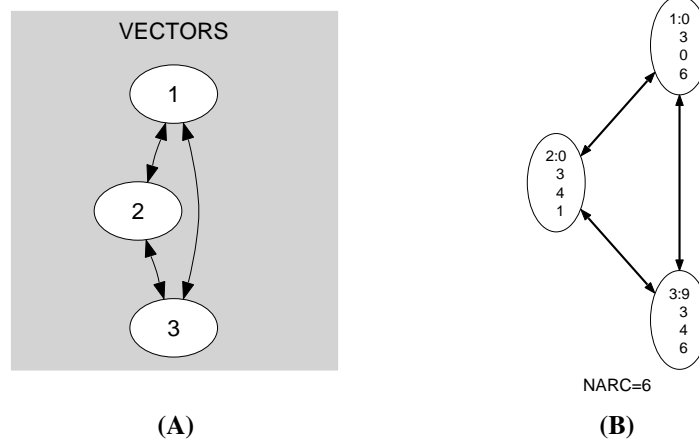

Figure 5.4: Initial and final graph of the all_differ_from_exactly_k_pos constraint

Signature

Since we use the $CLIQUE(\neq)$ arc generator on the items of the VECTORS collection, the expression |VECTORS| $\cdot$ |VECTORS| $-$ |VECTORS| corresponds to the maximum number of arcs of the final graph. Therefore we can rewrite the graph property **NARC** $=$ |VECTORS| $\cdot$ |VECTORS| $-$ |VECTORS| to **NARC** $\geq$ |VECTORS| $\cdot$ |VECTORS| $-$ |VECTORS|. This leads to simplify $\overline{\textbf{NARC}}$ to $\overline{\textbf{NARC}}$.