5.30 and

DESCRIPTION LINKS AUTOMATON

Origin

Logic

Constraint

and(VAR, VARIABLES)

Synonym

rel.

Arguments

VAR	:	dvar
VARIABLES	:	collection(var−dvar)

Restrictions

$\mathtt{VAR} \geq 0$
$\mathtt{VAR} \leq 1$
$|\mathtt{VARIABLES}| \geq 2$
required(VARIABLES, var)
$\mathtt{VARIABLES.var} \geq 0$
$\mathtt{VARIABLES.var} \leq 1$

Purpose

Let VARIABLES be a collection of 0-1 variables $\mathtt{VAR}_1, \mathtt{VAR}_2, \ldots, \mathtt{VAR}_n$ $(n \geq 2)$. Enforce $\mathtt{VAR} = \mathtt{VAR}_1 \wedge \mathtt{VAR}_2 \wedge \cdots \wedge \mathtt{VAR}_n$.

Example

$(0, \langle 0, 0\rangle)$
$(0, \langle 0, 1\rangle)$
$(0, \langle 1, 0\rangle)$
$(1, \langle 1, 1\rangle)$
$(0, \langle 1, 0, 1\rangle)$

Symmetry

Items of VARIABLES are permutable.

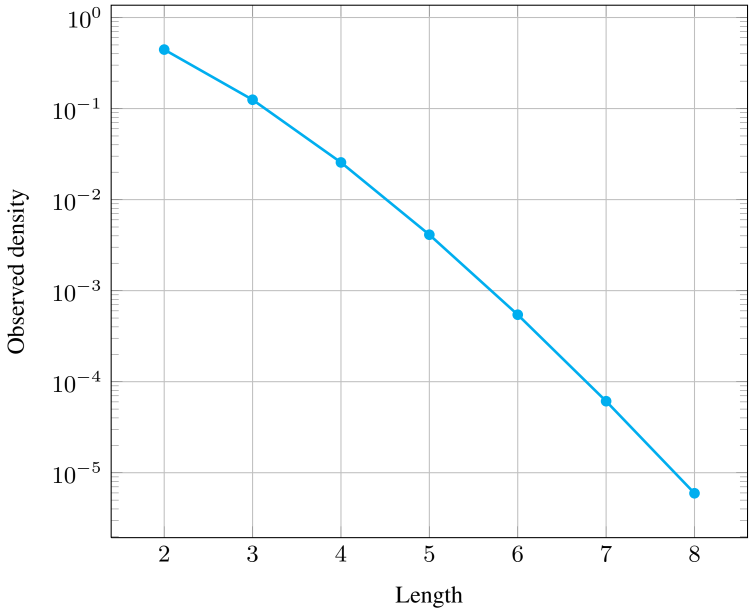
Arg. properties

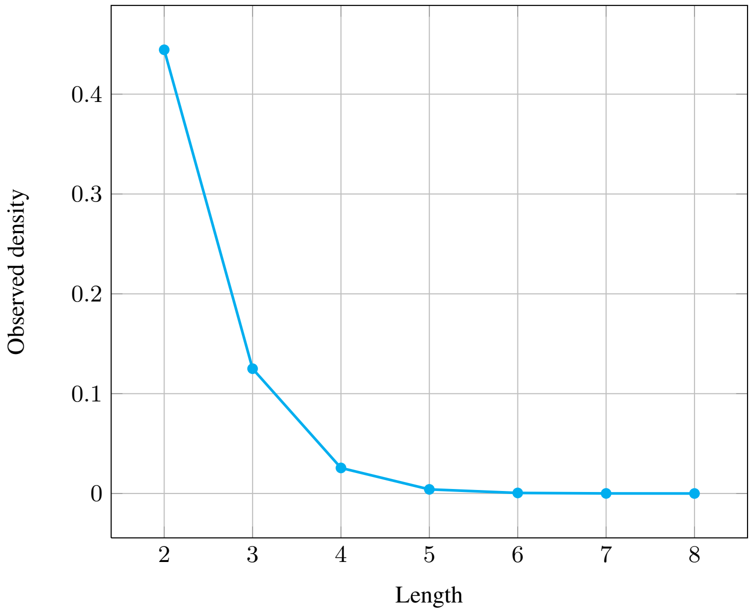
- Functional dependency: VAR determined by VARIABLES.
- Extensible wrt. VARIABLES when $\mathtt{VAR} = 0$.
- Aggregate: $\mathtt{VAR}(\wedge)$, $\mathtt{VARIABLES}(\mathtt{union})$.

Counting

Length (n)	2	3	4	5	6	7	8
Solutions	4	8	16	32	64	128	256

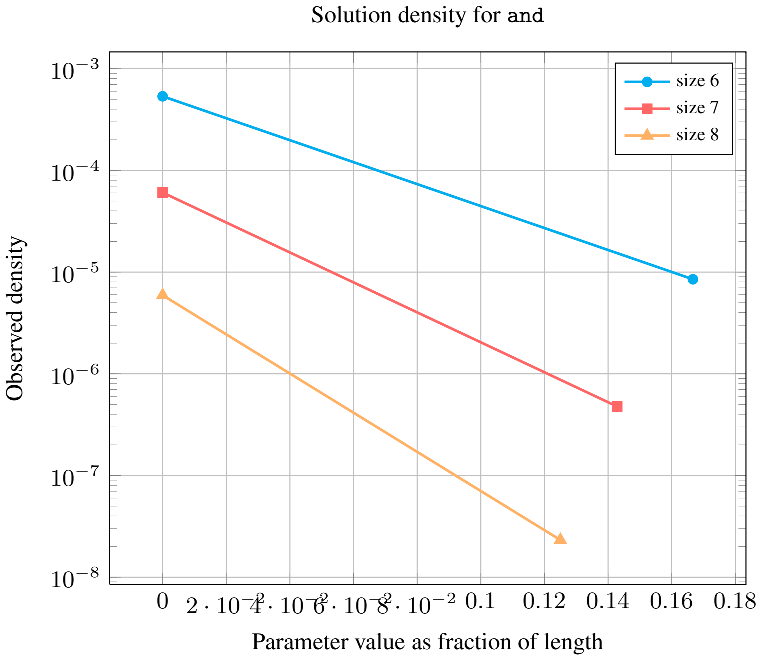
Number of solutions for and: domains $0..n$

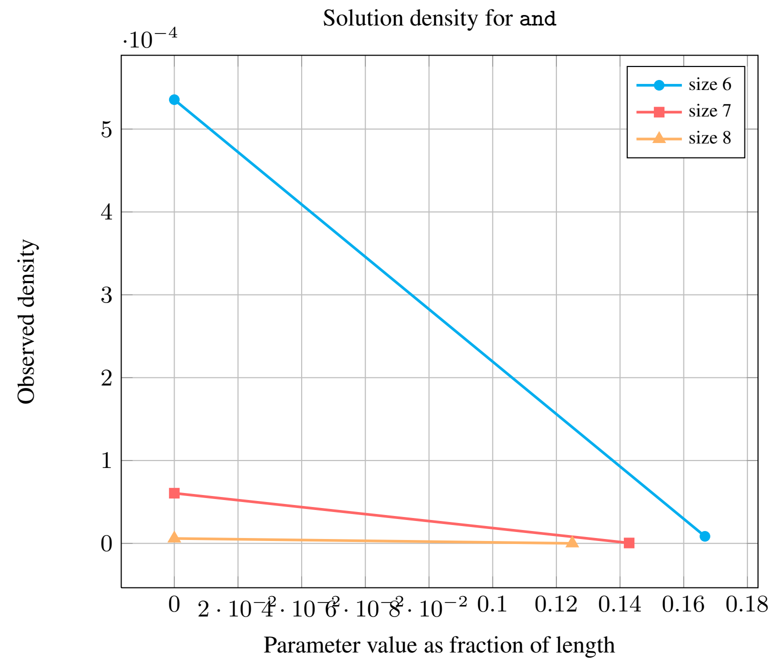

Solution density for `and`


Solution density for `and`

Length (n)		2	3	4	5	6	7	8
Total		4	8	16	32	64	128	256
Parameter value	0	3	7	15	31	63	127	255
	1	1	1	1	1	1	1	1

Solution count for `and`: domains $0..n$





Systems reifiedAnd in **Choco**, rel in **Gecode**, andbool in **JaCoP**, #/“in **SICStus**.

See also **common keyword:** `clause_and`, `equivalent`, `imply`, `nand`, `nor`, `or`, `xor` *(Boolean constraint)*.

implies: `atleast_nvalue`, `between_min_max`, `minimum`, `soft_all_equal_min_ctr`.

Keywords **characteristic of a constraint:** automaton, automaton without counters, reified automaton constraint.

constraint arguments: pure functional dependency.

constraint network structure: Berge-acyclic constraint network.

constraint type: Boolean constraint.

filtering: arc-consistency.

modelling: functional dependency.

Cond. implications

- $\texttt{and}(\texttt{VAR}, \texttt{VARIABLES})$
  with $|\texttt{VARIABLES}| > 2$
  implies $\texttt{some\_equal}(\texttt{VARIABLES})$.

- $\texttt{and}(\texttt{VAR}, \texttt{VARIABLES})$
  with $\texttt{VAR} = 0$
  implies $\texttt{nand}(\texttt{VAR}, \texttt{VARIABLES})$
  when $\texttt{VAR} = 1$.

- $\texttt{and}(\texttt{VAR}, \texttt{VARIABLES})$
  with $\texttt{VAR} = 1$
  implies $\texttt{nand}(\texttt{VAR}, \texttt{VARIABLES})$
  when $\texttt{VAR} = 0$.

Automaton Figure 5.68 depicts a first deterministic automaton without counter associated with the and constraint. To the first argument VAR of the and constraint corresponds the first signature variable. To each variable $\mathtt{VAR}_i$ of the second argument VARIABLES of the and constraint corresponds the next signature variable. There is no signature constraint.

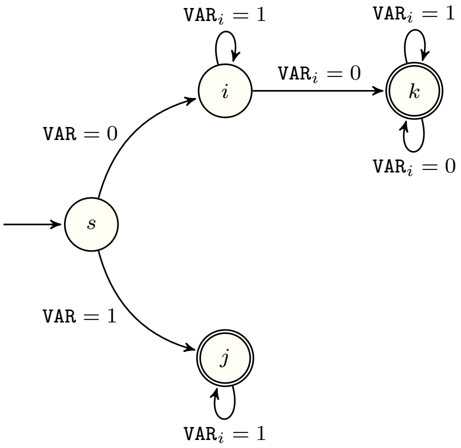


Figure 5.68: Counter free automaton of the $\mathtt{and}(\mathtt{VAR}, \langle \mathtt{VAR}_1, \mathtt{VAR}_2, \ldots, \mathtt{VAR}_n \rangle)$ constraint (the transition $i \xrightarrow{\mathtt{VAR}_i=0} k$ represents the fact that at least one variable $\mathtt{VAR}_i$ should be set to 0 when $\mathtt{VAR} = 0$, while the transition $j \xrightarrow{\mathtt{VAR}_i=1} j$ represents the fact that all $\mathtt{VAR}_i$ should be set to 1 when $\mathtt{VAR} = 1$)

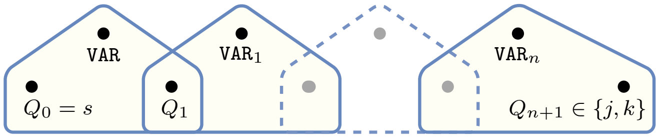


Figure 5.69: Hypergraph of the reformulation corresponding to the automaton of the and constraint

Figure 5.70 depicts a second deterministic automaton with one counter associated with the and constraint, where the argument VAR is unified to the final value of the counter.

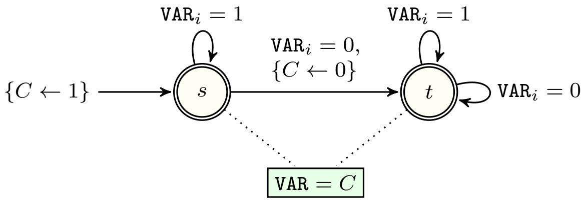


Figure 5.70: Automaton (with one counter) of the and constraint

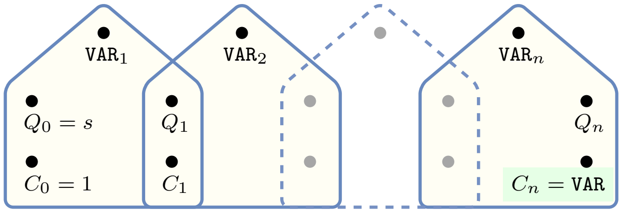


Figure 5.71: Hypergraph of the reformulation corresponding to the automaton (with one counter) of the `and` constraint (since all states of the automaton are accepting there is no restriction on the last variable Q_n)