5.36 atleast

DESCRIPTION LINKS GRAPH AUTOMATON

Origin CHIP

Constraint atleast(N, VARIABLES, VALUE)

Synonym count.

Arguments

N	:	int
VARIABLES	:	collection(var−dvar)
VALUE	:	int

Restrictions

$\mathtt{N} \geq 0$
$\mathtt{N} \leq |\mathtt{VARIABLES}|$
required(VARIABLES, var)

Purpose At least N variables of the VARIABLES collection are assigned value VALUE.

Example $(2, \langle 4, 2, 4, 5\rangle, 4)$

The atleast constraint holds since at least 2 values of the collection $\langle 4, 2, 4, 5\rangle$ are equal to value 4.

All solutions Figure 5.85 gives all solutions to the following non ground instance of the atleast constraint: $V_1 \in [3, 5]$, $V_2 \in [1, 2]$, $V_3 \in [5, 6]$, $V_4 \in [7, 9]$, $\mathtt{atleast}(\mathbf{2}, \langle V_1, V_2, V_3, V_4\rangle, \mathbf{5})$.

① $(\mathbf{2}, \langle \mathbf{5}, 1, \mathbf{5}, 7\rangle, \mathbf{5})$
② $(\mathbf{2}, \langle \mathbf{5}, 1, \mathbf{5}, 8\rangle, \mathbf{5})$
③ $(\mathbf{2}, \langle \mathbf{5}, 1, \mathbf{5}, 9\rangle, \mathbf{5})$
④ $(\mathbf{2}, \langle \mathbf{5}, 2, \mathbf{5}, 7\rangle, \mathbf{5})$
⑤ $(\mathbf{2}, \langle \mathbf{5}, 2, \mathbf{5}, 8\rangle, \mathbf{5})$
⑥ $(\mathbf{2}, \langle \mathbf{5}, 2, \mathbf{5}, 9\rangle, \mathbf{5})$

Figure 5.85: All solutions corresponding to the non ground example of the atleast constraint of the **All solutions** slot

Typical

$\mathtt{N} > 0$
$\mathtt{N} < |\mathtt{VARIABLES}|$
$|\mathtt{VARIABLES}| > 1$

Symmetries

- Items of VARIABLES are permutable.
- N can be decreased to any value ≥ 0.
- An occurrence of a value of VARIABLES.var that is different from VALUE can be replaced by any other value.

Arg. properties

Extensible wrt. VARIABLES.

Systems

occurenceMin in **Choco**, count in **Gecode**, atleast in **Gecode**, count in **JaCoP**, at_least in **MiniZinc**, count in **SICStus**.

Used in

alldifferent_except_0, among_diff_0, atmost, global_contiguity, int_value_precede, ith_pos_different_from_0, minimum_except_0, nvalues_except_0, period_except_0, sliding_card_skip0, weighted_partial_alldiff.

See also

common keyword: among (*value constraint*).

comparison swapped: atmost.

implied by: exactly ($\geq$ N *replaced by* $=$ N).

related: roots.

soft variant: open_atleast (*open constraint*).

Keywords

characteristic of a constraint: automaton, automaton with counters.

constraint network structure: alpha-acyclic constraint network(2).

constraint type: value constraint.

filtering: arc-consistency.

modelling: at least.

Arc input(s) VARIABLES

Arc generator $SELF \mapsto$ `collection`(`variables`)

Arc arity 1

Arc constraint(s) `variables.var` $=$ `VALUE`

Graph property(ies) $\mathbf{NARC} \geq$ `N`

Graph model Since each arc constraint involves only one vertex (`VALUE` is fixed), we employ the $SELF$ arc generator in order to produce a graph with a single loop on each vertex.

Parts (A) and (B) of Figure 5.86 respectively show the initial and final graph associated with the **Example** slot. Since we use the $\mathbf{NARC}$ graph property, the loops of the final graph are stressed in bold.

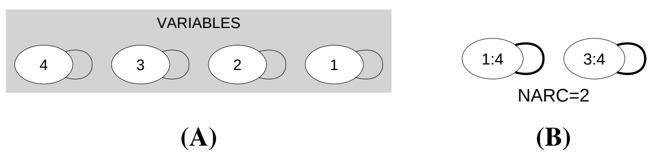

Figure 5.86: Initial and final graph of the `atleast` constraint

Automaton Figure 5.87 depicts the automaton associated with the `atleast` constraint. To each variable $\mathtt{VAR}_i$ of the collection `VARIABLES` corresponds a 0-1 signature variable S_i. The following signature constraint links $\mathtt{VAR}_i$ and S_i: $\mathtt{VAR}_i = \mathtt{VALUE} \Leftrightarrow S_i$. The automaton counts the number of variables of the `VARIABLES` collection that are assigned value `VALUE` and finally checks that this number is greater than or equal to `N`.

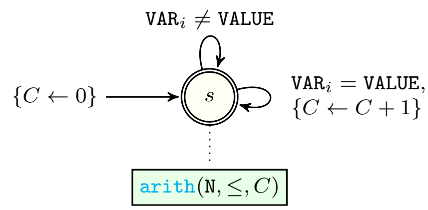

Figure 5.87: Automaton of the `atleast` constraint

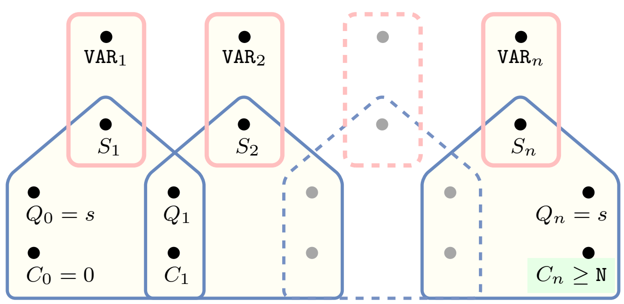

Figure 5.88: Hypergraph of the reformulation corresponding to the automaton (with one counter) of the `atleast` constraint: since all states variables $Q_0, Q_1, \ldots, Q_n$ are fixed to the unique state s of the automaton, the transitions constraints share only the counter variable C and the constraint network is Berge-acyclic