

5.37 `atleast_nvalue`

	DESCRIPTION	LINKS	GRAPH
Origin	[341]		
Constraint	<code>atleast_nvalue(NVAL, VARIABLES)</code>		
Synonym	<code>k_diff</code> .		
Arguments	NVAL : <code>dvar</code> VARIABLES : <code>collection(var-dvar)</code>		
Restrictions	<code>required(VARIABLES, var)</code> $NVAL \geq 0$ $NVAL \leq VARIABLES $ $NVAL \leq \text{range}(VARIABLES.var)$		
Purpose	<div style="border: 1px solid pink; padding: 5px;"> The number of distinct values taken by the variables of the collection <code>VARIABLES</code> is greater than or equal to <code>NVAL</code>. </div>		
Example	<div style="border: 1px solid blue; padding: 5px;"> $(2, \langle 3, 1, 7, 1, 6 \rangle)$ $(4, \langle 3, 1, 7, 1, 6 \rangle)$ $(5, \langle 3, 1, 7, 0, 6 \rangle)$ </div> <p>The first <code>atleast_nvalue</code> constraint holds since the collection $\langle 3, 1, 7, 1, 6 \rangle$ involves at least 2 distinct values (i.e., in fact 4 distinct values).</p>		
Typical	$NVAL > 0$ $NVAL < VARIABLES $ $NVAL < \text{range}(VARIABLES.var)$ $ VARIABLES > 1$		
Symmetries	<ul style="list-style-type: none"> • <code>NVAL</code> can be <code>decreased</code> to any value ≥ 0. • Items of <code>VARIABLES</code> are <code>permutable</code>. • All occurrences of two distinct values of <code>VARIABLES.var</code> can be <code>swapped</code>; all occurrences of a value of <code>VARIABLES.var</code> can be <code>renamed</code> to any unused value. 		
Arg. properties	<code>Extensible</code> wrt. <code>VARIABLES</code> .		
Remark	The <code>atleast_nvalue</code> constraint was first introduced by J.-C. Régin under the name <code>k_diff</code> in [341]. Later on the <code>atleast_nvalue</code> constraint was introduced together with the <code>atmost_nvalue</code> constraint by C. Bessière <i>et al.</i> in an article [62] providing filtering algorithms for the <code>nvalue</code> constraint.		

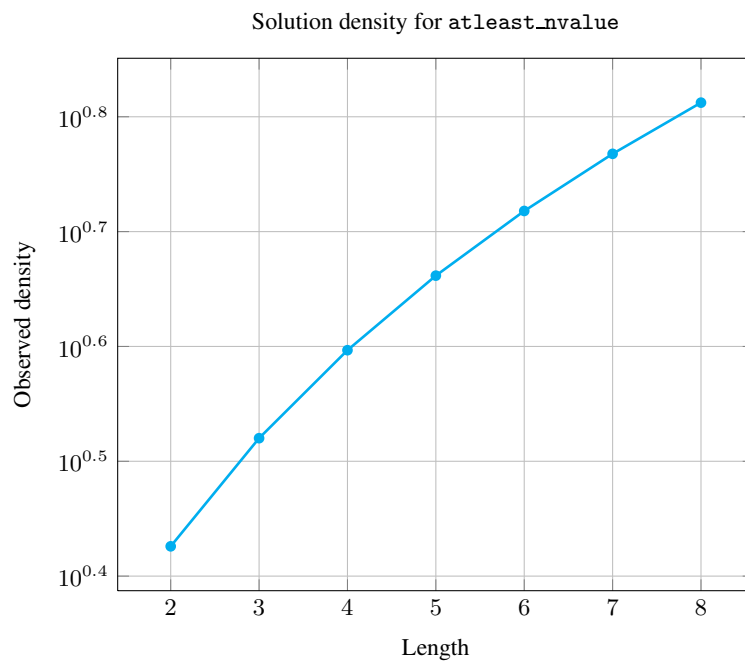
Algorithm

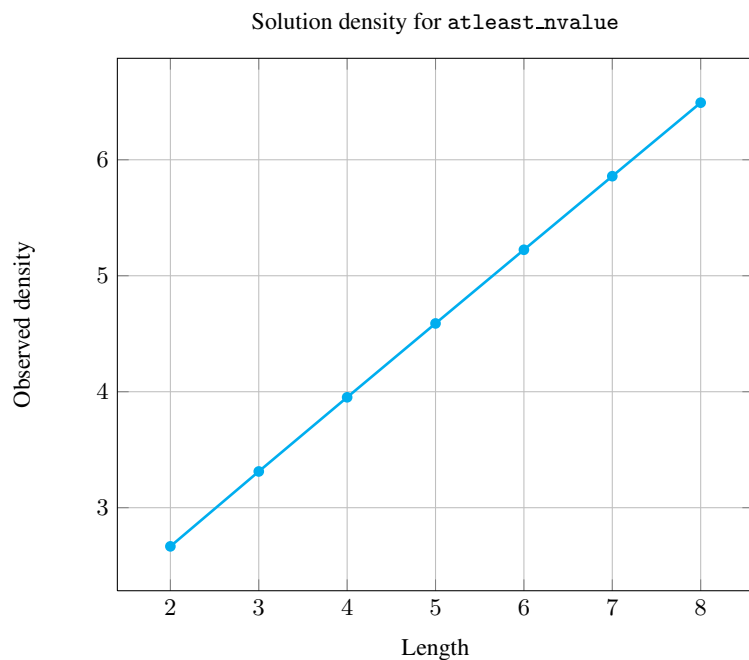
[62] provides a sketch of a filtering algorithm enforcing [arc-consistency](#) for the `atleast_nvalue` constraint. This algorithm is based on the maximal matching in a bipartite graph.

Counting

Length (n)	2	3	4	5	6	7	8
Solutions	24	212	2470	35682	614600	12286024	279472266

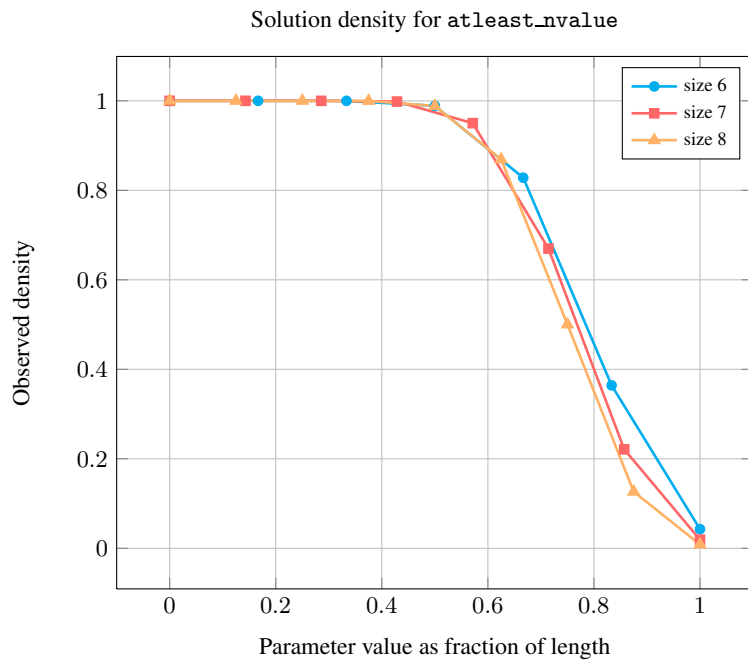
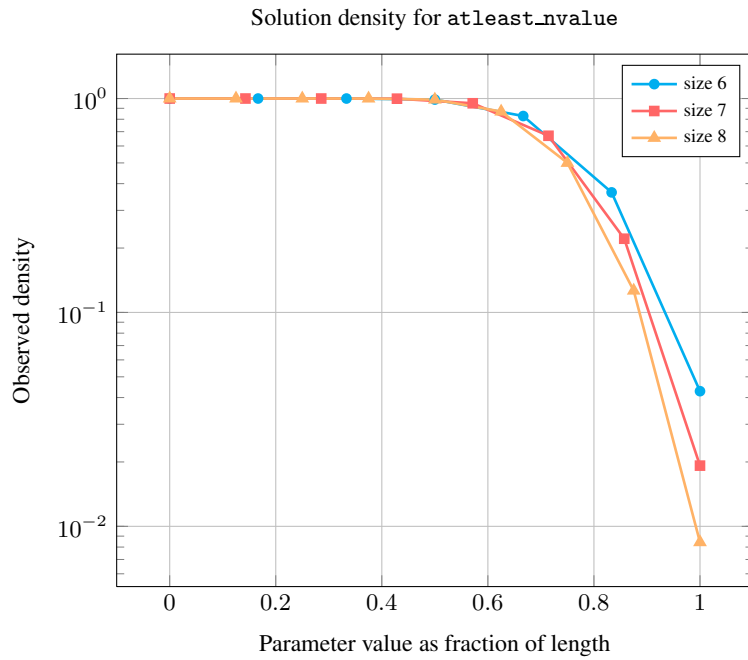
Number of solutions for `atleast_nvalue`: domains $0..n$





Length (n)		2	3	4	5	6	7	8
Total		24	212	2470	35682	614600	12286024	279472266
Parameter value	0	9	64	625	7776	117649	2097152	43046721
	1	9	64	625	7776	117649	2097152	43046721
	2	6	60	620	7770	117642	2097144	43046712
	3	-	24	480	7320	116340	2093616	43037568
	4	-	-	120	4320	97440	1992480	42550704
	5	-	-	-	720	42840	1404480	37406880
	6	-	-	-	-	5040	463680	21530880
	7	-	-	-	-	-	40320	5443200
	8	-	-	-	-	-	-	362880

Solution count for atleast_nvalue: domains 0.. n



See also

[comparison swapped: atmost_nvalue.](#)

[implied by: and, equivalent, imply, nand, nor, nvalue](#) (\geq NVAL replaced by = NVAL), [nvisible_from_end, nvisible_from_start, or, size_max_seq_alldifferent,](#)

`size_max_starting_seq_alldifferent`, `xor`.

uses in its reformulation: `not_all_equal`.

Keywords

constraint type: counting constraint, value partitioning constraint.

filtering: bipartite matching, arc-consistency.

final graph structure: strongly connected component, equivalence.

modelling: number of distinct equivalence classes, number of distinct values.

Arc input(s)	VARIABLES
Arc generator	<i>CLIQUE</i> \mapsto collection(variables1, variables2)
Arc arity	2
Arc constraint(s)	variables1.var = variables2.var
Graph property(ies)	NSCC \geq NVAL
Graph class	EQUIVALENCE

Graph model

Parts (A) and (B) of Figure 5.89 respectively show the initial and final graph associated with the first example of the **Example** slot. Since we use the **NSCC** graph property we show the different strongly connected components of the final graph. Each strongly connected component corresponds to a specific value that is assigned to some variables of the VARIABLES collection. The 4 following values 1, 3, 6 and 7 are used by the variables of the VARIABLES collection.

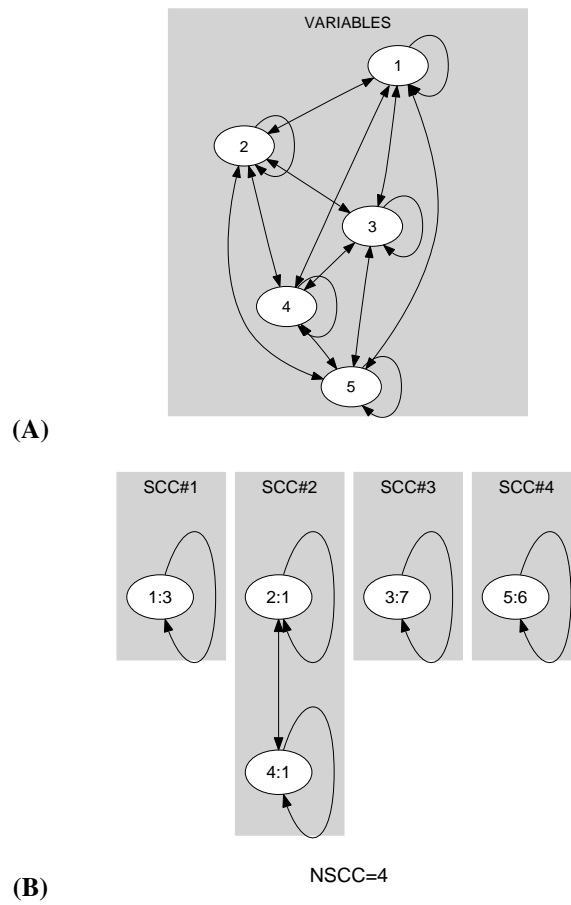


Figure 5.89: Initial and final graph of the atleast_nvalue constraint

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