5.47   balance_partition

Origin   Derived from balance.

Constraint   balance_partition(BALANCE, VARIABLES, PARTITIONS)

Type   VALUES   :   collection(val−int)

Arguments   BALANCE   :   dvar
                        VARIABLES   :   collection(var−dvar)
                        PARTITIONS :   collection(p − VALUES)

Restrictions $|\text{VALUES}| \geq 1$
                        required(VALUES, val)
                        distinct(VALUES, val)
                        $\text{BALANCE} \geq 0$
                        $\text{BALANCE} \leq \max(0, |\text{VARIABLES}| - 2)$
                        required(VARIABLES, var)
                        required(PARTITIONS, p)
                        $|\text{PARTITIONS}| \geq 2$

Purpose   Consider the largest set $\mathcal{S}_1$ (respectively the smallest set $\mathcal{S}_2$) of variables of the collection VARIABLES that take their value in the same partition of the collection PARTITIONS.BALANCE is equal to the difference between the cardinality of $\mathcal{S}_2$ and the cardinality of $\mathcal{S}_1$.

Example $(1, \langle 6, 2, 6, 4, 4 \rangle, \langle \text{p} − \langle 1, 3 \rangle, \text{p} − \langle 4 \rangle, \text{p} − \langle 2, 6 \rangle \rangle)$

                        In this example values $6, 2, 6, 4, 4$ are respectively associated with the partitions $\text{p} − \langle 2, 6 \rangle$ and $\text{p} − \langle 4 \rangle$. Partitions $\text{p} − \langle 4 \rangle$ and $\text{p} − \langle 2, 6 \rangle$ are respectively used 2 and 3 times. The balance_partition constraint holds since its first argument BALANCE is assigned to the difference between the maximum and minimum number of the previous occurrences (i.e., $3 − 2$). Note that we do not consider those partitions that are not used at all.

Typical $|\text{VARIABLES}| > 2$
                        $|\text{VARIABLES}| > |\text{PARTITIONS}|$

Symmetries   • Items of VARIABLES are permutable.

                        • Items of PARTITIONS are permutable.

                        • Items of PARTITIONS.p are permutable.

                        • An occurrence of a value of VARIABLES.var can be replaced by any other value that also belongs to the same partition of PARTITIONS.

Arg. properties

Functional dependency: BALANCE determined by VARIABLES and PARTITIONS.

Usage

An application of the balance_partition is to enforce a *balanced assignment* of values, no matter how many distinct partitions will be used. In this case one will *push down* the maximum value of the first argument of the balance_partition constraint.

See also

specialisation: balance *(*variable ∈ partition *replaced by* variable*)*.

used in graph description: in_same_partition.

Keywords

application area: assignment.

characteristic of a constraint: partition.

constraint arguments: pure functional dependency.

constraint type: value constraint.

final graph structure: equivalence.

modelling: balanced assignment, functional dependency.

Arc input(s)	VARIABLES
Arc generator	$CLIQUE \mapsto$ collection(variables1, variables2)
Arc arity	2
Arc constraint(s)	in_same_partition(variables1.var, variables2.var, PARTITIONS)
Graph property(ies)	**RANGE_NSCC**= BALANCE
Graph class	EQUIVALENCE

Graph model

The graph property **RANGE_NSCC** constraints the difference between the sizes of the largest and smallest strongly connected components.

Parts (A) and (B) of Figure 5.107 respectively show the initial and final graph associated with the **Example** slot. Since we use the **RANGE_NSCC** graph property, we show the largest and smallest strongly connected components of the final graph.
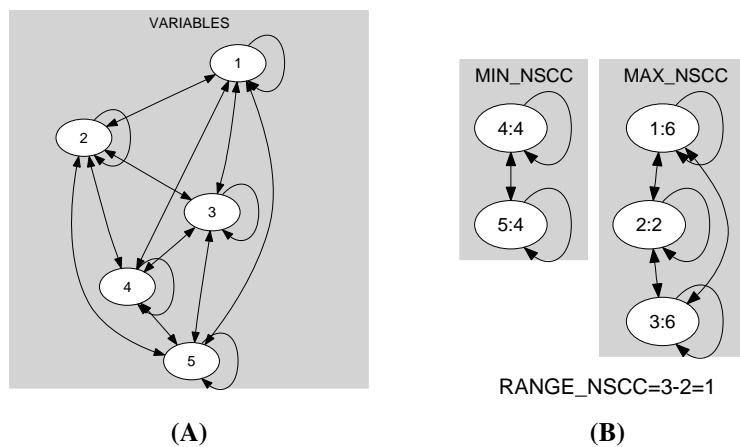

(A)   **(B)**

Figure 5.107: Initial and final graph of the `balance_partition` constraint