5.71 clique

DESCRIPTION LINKS GRAPH

Origin [159]

Constraint `clique(SIZE_CLIQUE, NODES)`

Arguments

```
SIZE_CLIQUE  :  dvar
NODES        :  collection(index-int, succ-svar)
```

Restrictions

$\texttt{SIZE\_CLIQUE} \geq 0$
$\texttt{SIZE\_CLIQUE} \leq |\texttt{NODES}|$
$\texttt{required}(\texttt{NODES}, [\texttt{index}, \texttt{succ}])$
$\texttt{NODES.index} \geq 1$
$\texttt{NODES.index} \leq |\texttt{NODES}|$
$\texttt{distinct}(\texttt{NODES}, \texttt{index})$
$\texttt{NODES.succ} \geq 1$
$\texttt{NODES.succ} \leq |\texttt{NODES}|$

Purpose

Consider a digraph G described by the NODES collection: to the i^{th} item of the NODES collection corresponds the i^{th} vertex of G; To each value j of the i^{th} succ variable corresponds an arc from the i^{th} vertex to the j^{th} vertex. Select a subset $\mathcal{S}$ of the vertices of G that forms a clique of size SIZE_CLIQUE (i.e., there is an arc between each pair of distinct vertices of $\mathcal{S}$).

Example

$$\left(3, \left\langle \begin{array}{ll} \texttt{index}-1 & \texttt{succ}-\emptyset, \\ \texttt{index}-2 & \texttt{succ}-\{3,5\}, \\ \texttt{index}-3 & \texttt{succ}-\{2,5\}, \\ \texttt{index}-4 & \texttt{succ}-\emptyset, \\ \texttt{index}-5 & \texttt{succ}-\{2,3\} \end{array} \right\rangle \right)$$

The `clique` constraint holds since the NODES collection depicts a clique involving 3 vertices (namely vertices 2, 3 and 5) and since its first argument SIZE_CLIQUE is set to the number of vertices of this clique.

Typical

$\texttt{SIZE\_CLIQUE} \geq 2$
$\texttt{SIZE\_CLIQUE} < |\texttt{NODES}|$
$|\texttt{NODES}| > 2$

Symmetry Items of NODES are permutable.

Arg. properties

Functional dependency: SIZE_CLIQUE determined by NODES.

Algorithm [159], [347, 348]. The algorithm for finding maximum cliques in an undirected graph of C. Bron and J. Kerbosch [88] was adapted by J.-C. Régin to the context of constraint programming in his papers.

See also **common keyword:** link_set_to_booleans (*constraint involving set variables, can be used for channelling*).

used in graph description: in_set.

Keywords **constraint arguments:** constraint involving set variables.

constraint type: graph constraint.

final graph structure: symmetric.

modelling: functional dependency.

problems: maximum clique.

Arc input(s)	NODES
Arc generator	$CLIQUE(\neq) \mapsto$collection(nodes1, nodes2)
Arc arity	2
Arc constraint(s)	in_set(nodes2.index, nodes1.succ)
Graph property(ies)	• **NARC**= SIZE_CLIQUE $*$ SIZE_CLIQUE $-$ SIZE_CLIQUE • **NVERTEX**= SIZE_CLIQUE
Graph class	SYMMETRIC

Graph model Note the use of *set variables* for modelling the fact that the vertices of the final graph have more than one successor: The successor variable associated with each vertex contains the successors of the corresponding vertex.

Part (A) of Figure 5.175 shows the initial graph from which we start. It is derived from the set associated with each vertex. Each set describes the potential values of the succ attribute of a given vertex. Part (B) of Figure 5.175 gives the final graph associated with the **Example** slot. Since we both use the **NARC** and **NVERTEX** graph properties, the arcs and the vertices of the final graph are stressed in bold. The final graph corresponds to a clique containing three vertices.

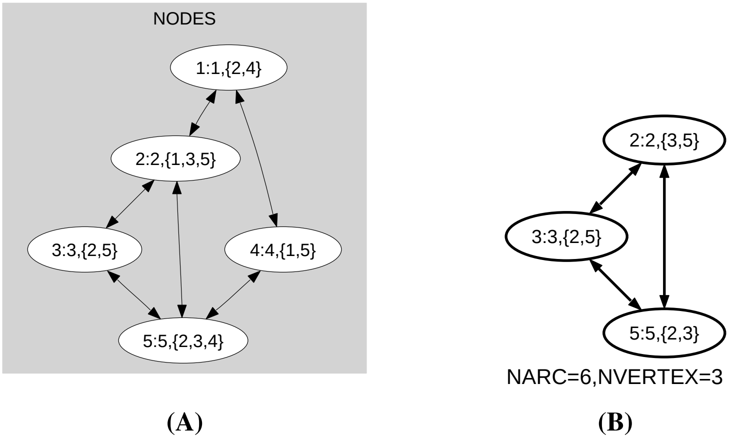

Figure 5.175: Initial and final graph of the clique set constraint