5.95 crossing

DESCRIPTION LINKS GRAPH

Origin Inspired by [122].

Constraint crossing(NCROSS, SEGMENTS)

Arguments

NCROSS	:	dvar
SEGMENTS	:	collection(ox−dvar, oy−dvar, ex−dvar, ey−dvar)

Restrictions

NCROSS ≥ 0
NCROSS $\leq$ (|SEGMENTS| $*$ |SEGMENTS| $-$ |SEGMENTS|)/2
required(SEGMENTS, [ox, oy, ex, ey])

Purpose NCROSS is the number of line segments intersections between the line segments defined by the SEGMENTS collection. Each line segment is defined by the coordinates (ox, oy) and (ex, ey) of its two extremities.

Example

$$\left(3, \left\langle \begin{array}{cccc} \mathtt{ox}-1 & \mathtt{oy}-4 & \mathtt{ex}-9 & \mathtt{ey}-2, \\ \mathtt{ox}-1 & \mathtt{oy}-1 & \mathtt{ex}-3 & \mathtt{ey}-5, \\ \mathtt{ox}-3 & \mathtt{oy}-2 & \mathtt{ex}-7 & \mathtt{ey}-4, \\ \mathtt{ox}-9 & \mathtt{oy}-1 & \mathtt{ex}-9 & \mathtt{ey}-4 \end{array} \right\rangle\right)$$

Figure 5.211 provides a picture of the example with the corresponding four line segments of the SEGMENTS collection. The crossing constraint holds since its first argument NCROSS is set to 3, which is actually the number of line segments intersections.

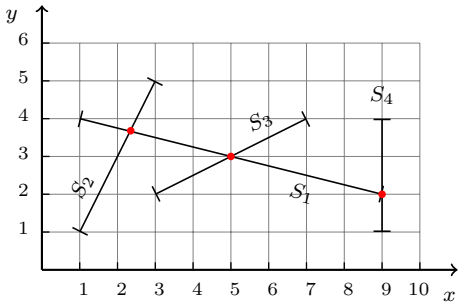

Figure 5.211: Illustration of the **Example** slot: intersection, in red, between the four line segments S_1, S_2, S_3 and S_4 (NCROSS $= 3$)

Typical |SEGMENTS| > 1

Symmetries

- Items of SEGMENTS are permutable.
- Attributes of SEGMENTS are permutable w.r.t. permutation (ox, oy) (ex, ey) (*permutation applied to all items*).
- One and the same constant can be added to the ox and ex attributes of all items of SEGMENTS.
- One and the same constant can be added to the oy and ey attributes of all items of SEGMENTS.

Arg. properties

Functional dependency: NCROSS determined by SEGMENTS.

See also

common keyword: graph_crossing, two_layer_edge_crossing (*line segments intersection*).

Keywords

constraint arguments: pure functional dependency.

final graph structure: acyclic, no loop.

geometry: geometrical constraint, line segments intersection.

modelling: functional dependency.

Arc input(s)	SEGMENTS
Arc generator	$CLIQUE(<) \mapsto \texttt{collection}(\texttt{s1}, \texttt{s2})$
Arc arity	2
Arc constraint(s)	• $\texttt{max}(\texttt{s1.ox}, \texttt{s1.ex}) \geq \texttt{min}(\texttt{s2.ox}, \texttt{s2.ex})$ • $\texttt{max}(\texttt{s2.ox}, \texttt{s2.ex}) \geq \texttt{min}(\texttt{s1.ox}, \texttt{s1.ex})$ • $\texttt{max}(\texttt{s1.oy}, \texttt{s1.ey}) \geq \texttt{min}(\texttt{s2.oy}, \texttt{s2.ey})$ • $\texttt{max}(\texttt{s2.oy}, \texttt{s2.ey}) \geq \texttt{min}(\texttt{s1.oy}, \texttt{s1.ey})$ • $\bigvee \left(\begin{array}{ll} (\texttt{s2.ox} - \texttt{s1.ex}) * (\texttt{s1.ey} - \texttt{s1.oy}) - & \\ (\texttt{s1.ex} - \texttt{s1.ox}) * (\texttt{s2.oy} - \texttt{s1.ey}) & = 0, \\ (\texttt{s2.ex} - \texttt{s1.ex}) * (\texttt{s2.oy} - \texttt{s1.oy}) - & \\ (\texttt{s2.ox} - \texttt{s1.ox}) * (\texttt{s2.ey} - \texttt{s1.ey}) & = 0, \\ \texttt{sign} \left(\begin{array}{l} (\texttt{s2.ox} - \texttt{s1.ex}) * (\texttt{s1.ey} - \texttt{s1.oy}) - \\ (\texttt{s1.ex} - \texttt{s1.ox}) * (\texttt{s2.oy} - \texttt{s1.ey}) \end{array} \right) & \neq \\ \texttt{sign} \left(\begin{array}{l} (\texttt{s2.ex} - \texttt{s1.ex}) * (\texttt{s2.oy} - \texttt{s1.oy}) - \\ (\texttt{s2.ox} - \texttt{s1.ox}) * (\texttt{s2.ey} - \texttt{s1.ey}) \end{array} \right) & \end{array} \right)$
Graph property(ies)	NARC= NCROSS
Graph class	• ACYCLIC • NO_LOOP

Graph model Each line segment is described by the x and y coordinates of its two extremities. In the arc generator we use the restriction $<$ in order to generate a single arc for each pair of segments. This is required, since otherwise we would count more than once a given line segments intersection.

Parts (A) and (B) of Figure 5.212 respectively show the initial and final graph associated with the **Example** slot. Since we use the NARC graph property, the arcs of the final graph are stressed in bold. An arc constraint expresses the fact the two line segments intersect. It is taken from [122, page 889]. Each arc of the final graph corresponds to a line segments intersection.

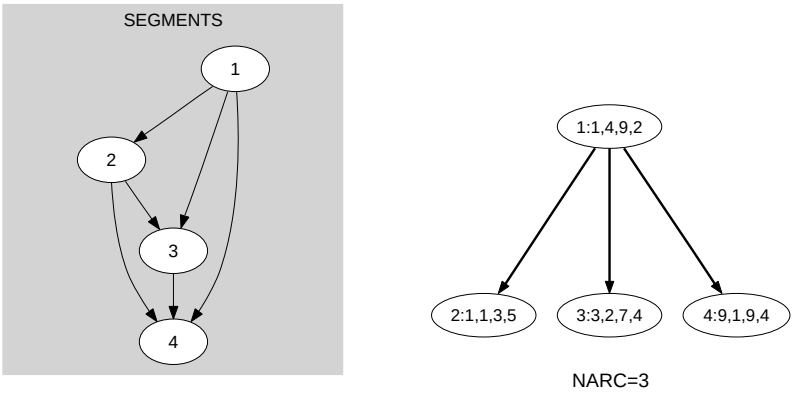

Figure 5.212: Initial and final graph of the `crossing` constraint