5.9	99 cumulative_two_d
	DESCRIPTION LINKS
Origin	Inspired by cumulative and diffn.
Constraint	cumulative_two_d(RECTANGLES,LIMIT)
Arguments	RECTANGLES : collection $\begin{pmatrix} start1-dvar, \\ size1-dvar, \\ last1-dvar, \\ start2-dvar, \\ size2-dvar, \\ last2-dvar, \\ height-dvar \end{pmatrix}$ LIMIT : int
Restrictions Purpose	$\begin{array}{l} \hline \textbf{require_at_least(2, \text{RECTANGLES}, [start1, size1, last1])} \\ \hline \textbf{require_at_least(2, \text{RECTANGLES}, [start2, size2, last2])} \\ \hline \textbf{required}(\text{RECTANGLES}, \text{height}) \\ \hline \textbf{RECTANGLES}.size1 \geq 0 \\ \hline \textbf{RECTANGLES}.size2 \geq 0 \\ \hline \textbf{RECTANGLES}.height \geq 0 \\ \hline \textbf{LIMIT} \geq 0 \\ \end{array}$ Consider a set \$\mathcal{R}\$ of rectangles described by the RECTANGLES collection. Enforces that at each point of the plane, the cumulated height of the set of rectangles that overlap that point, does not exceed a given limit.
Example	$\left(\begin{array}{c ccccccccccccccccccccccccccccccccccc$
	Part (A) of Figure 5.222 shows the 4 parallelepipeds of height 4, 2, 3 and 1 associated with the items of the RECTANGLES collection (parallelepipeds since each rectangle also has a height). Part (B) gives the corresponding cumulated 2-dimensional profile, where each number is the cumulated height of all the rectangles that contain the corresponding region. The cumulative_two_d constraint holds since the highest peak of the cumulated 2-dimensional profile does not exceed the upper limit 4 imposed by the last argument of the cumulative_two_d constraint.
Typical	<pre> RECTANGLES > 1 RECTANGLES.size1 > 0 RECTANGLES.size2 > 0 RECTANGLES.height > 0 LIMIT <sum(rectangles.height)< pre=""></sum(rectangles.height)<></pre>

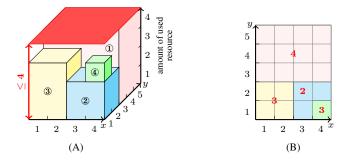


Figure 5.222: Two representations of a 2-dimensional cumulative profile of the **Example** slot (where the profile provides for each point of coordinates (c_x, c_y) the corresponding sum of the heights of the items intersecting that point): (A) a three dimensional representation and (B) a two dimensional representation from above with the height of the profile in red; as for the cumulative constraint the position of an item on the z axis does not matter, i.e. only its height matters.

Symmetries	• Items of RECTANGLES are permutable.
	• Attributes of RECTANGLES are permutable w.r.t. permutation (start1, start2) (size1, size2) (last1, last2) (height) (permutation applied to all items).
	• RECTANGLES.height can be decreased to any value ≥ 0 .
	• One and the same constant can be added to the start1 and last1 attributes of all items of RECTANGLES.
	• One and the same constant can be added to the start2 and last2 attributes of all items of RECTANGLES.
	• LIMIT can be increased.
Arg. properties	Contractible wrt. RECTANGLES.
Usage	The cumulative_two_d constraint is a necessary condition for the diffn constraint in 3 dimensions (i.e., the placement of parallelepipeds in such a way that they do not pairwise overlap and that each parallelepiped has his sides parallel to the sides of the placement space).
Algorithm	A first natural way to handle this constraint would be to accumulate the compulsory part [250] of the different rectangles in a quadtree [367]. To each leave of the quadtree we associate the cumulated height of the rectangles containing the corresponding region.
Systems	geost in Choco.
See also	related: diffn(cumulative_two_d is a necessary condition for diffn: forget one di- mension when the number of dimensions is equal to 3).
	specialisation: bin_packing(square of size 1 with a height replaced by task of duration 1), cumulative(rectangle with a height replaced by task with same height).

Keywords

characteristic of a constraint: derived collection. constraint type: predefined constraint. filtering: quadtree, compulsory part. geometry: geometrical constraint.