5.100 cumulative_with_level_of_priority

DESCRIPTION LINKS GRAPH

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Constraint cumulative_with_level_of_priority(TASKS, PRIORITIES)

Arguments

$$\begin{array}{lll} \texttt{TASKS} & : & \texttt{collection}\left(\begin{array}{l}\texttt{priority}-\texttt{int},\\ \texttt{origin}-\texttt{dvar},\\ \texttt{duration}-\texttt{dvar},\\ \texttt{end}-\texttt{dvar},\\ \texttt{height}-\texttt{dvar}\end{array}\right)\\ \texttt{PRIORITIES} & : & \texttt{collection}(\texttt{id}-\texttt{int},\texttt{capacity}-\texttt{int})\end{array}$$

Restrictions

required(TASKS, [priority, height])
require_at_least(2, TASKS, [origin, duration, end])
TASKS.priority ≥ 1
TASKS.priority $\leq$ |PRIORITIES|
TASKS.duration ≥ 0
TASKS.origin $\leq$ TASKS.end
TASKS.height ≥ 0
required(PRIORITIES, [id, capacity])
PRIORITIES.id ≥ 1
PRIORITIES.id $\leq$ |PRIORITIES|
increasing_seq(PRIORITIES, id)
increasing_seq(PRIORITIES, capacity)

Purpose

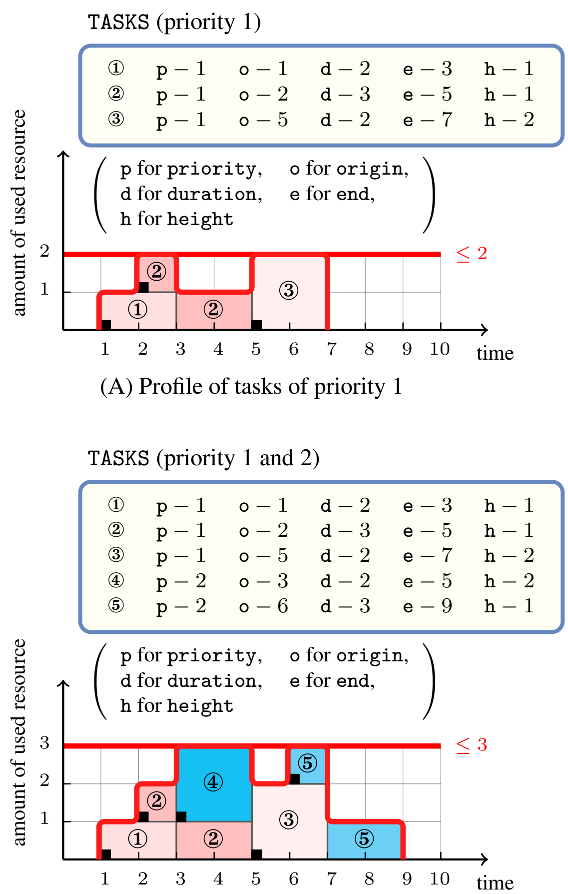
Consider a set $\mathcal{T}$ of tasks described by the TASKS collection where each task has a given priority chosen in the range $[1, \texttt{PRIORITIES}]$. Let $\mathcal{T}_i$ denote the subset of tasks of $\mathcal{T}$ that all have a priority less than or equal to i. For each set $\mathcal{T}_i$, the cumulative_with_level_of_priority constraint forces that at each point in time, the cumulated height of the set of tasks that overlap that point, does not exceed a given limit. A task overlaps a point i if and only if (1) its origin is less than or equal to i, and (2) its end is strictly greater than i. Finally, it also imposes for each task of $\mathcal{T}$ the constraint origin + duration = end.

Example

$$\left(\begin{array}{l}\left\langle\begin{array}{lllll}\texttt{priority}-1 & \texttt{origin}-1 & \texttt{duration}-2 & \texttt{end}-3 & \texttt{height}-1,\\ \texttt{priority}-1 & \texttt{origin}-2 & \texttt{duration}-3 & \texttt{end}-5 & \texttt{height}-1,\\ \texttt{priority}-1 & \texttt{origin}-5 & \texttt{duration}-2 & \texttt{end}-7 & \texttt{height}-2,\\ \texttt{priority}-2 & \texttt{origin}-3 & \texttt{duration}-2 & \texttt{end}-5 & \texttt{height}-2,\\ \texttt{priority}-2 & \texttt{origin}-6 & \texttt{duration}-3 & \texttt{end}-9 & \texttt{height}-1\end{array}\right\rangle,\\ \langle\texttt{id}-1\ \texttt{capacity}-2, \texttt{id}-2\ \texttt{capacity}-3\rangle\end{array}\right)$$

Figure 5.223 shows the cumulated profile associated with both levels of priority. To each task of the cumulative_with_level_of_priority constraint corresponds a set of rectangles containing the same number (i.e., the position of the task within the TASKS collection): the sum of the lengths of the rectangles corresponds to the duration of the task, while the height of the rectangles (i.e., all the rectangles associated with a task have the same height) corresponds to the resource consumption of the task. Tasks that have a priority of 1 are coloured in pink, while tasks that have a priority of 2 are coloured in blue. The `cumulative_with_level_of_priority` constraint holds since:

- At each point in time the cumulated resource consumption profile of the tasks of priority 1 does not exceed the upper capacity 2 enforced by the first item of the `PRIORITIES` collection.
- At each point in time the cumulated resource consumption profile of the tasks of priority 1 and 2 does not exceed the upper capacity 3 enforced by the second item of the `PRIORITIES` collection.



(A) Profile of tasks of priority 1

(B) Profile of tasks of priority 1 and 2

Figure 5.223: Resource consumption profiles according to both levels of priority for the tasks of the **Example** slot

Typical

$|\mathtt{TASKS}| > 1$
$\mathtt{range}(\mathtt{TASKS.priority}) > 1$
$\mathtt{range}(\mathtt{TASKS.origin}) > 1$
$\mathtt{range}(\mathtt{TASKS.duration}) > 1$
$\mathtt{range}(\mathtt{TASKS.end}) > 1$
$\mathtt{range}(\mathtt{TASKS.height}) > 1$
$\mathtt{TASKS.duration} > 0$
$\mathtt{TASKS.height} > 0$
$|\mathtt{PRIORITIES}| > 1$
$\mathtt{PRIORITIES.capacity} > 0$
$\mathtt{PRIORITIES.capacity} < \mathtt{sum}(\mathtt{TASKS.height})$
$|\mathtt{TASKS}| > |\mathtt{PRIORITIES}|$

Symmetries

- Items of `TASKS` are permutable.
- `TASKS.priority` can be increased to any value $\leq |\mathtt{PRIORITIES}|$.
- `TASKS.height` can be decreased to any value ≥ 0.
- One and the same constant can be added to the `origin` and `end` attributes of all items of `TASKS`.
- `PRIORITIES.capacity` can be increased.

Arg. properties

Contractible wrt. `TASKS`.

Usage

The `cumulative_with_level_of_priority` constraint was suggested by problems from the telecommunication area where one has to ensure different levels of quality of service. For this purpose the capacity of a transmission link is split so that a given percentage is reserved to each level. In addition we have that, if the capacities allocated to levels $1, 2, \ldots, i$ is not completely used, then level $i+1$ can use the corresponding spare capacity.

Remark

The `cumulative_with_level_of_priority` constraint can be modelled by a conjunction of `cumulative` constraints. As shown by the next example, the consistency for all variables of the `cumulative` constraints does not implies consistency for the corresponding `cumulative_with_level_of_priority` constraint. The following `cumulative_with_level_of_priority` constraint

$$\left(\begin{array}{l} \left\langle\begin{array}{llll} \mathtt{priority}-1 & \mathtt{origin}-o_1 & \mathtt{duration}-2 & \mathtt{height}-2, \\ \mathtt{priority}-1 & \mathtt{origin}-o_2 & \mathtt{duration}-2 & \mathtt{height}-1, \\ \mathtt{priority}-2 & \mathtt{origin}-o_3 & \mathtt{duration}-1 & \mathtt{height}-3 \end{array}\right\rangle, \\ \left\langle\begin{array}{ll} \mathtt{id}-1 & \mathtt{capacity}-2, \\ \mathtt{id}-2 & \mathtt{capacity}-3 \end{array}\right\rangle \end{array}\right)$$

where the domains of o_1, o_2 and o_3 are respectively equal to $\{1, 2, 3\}$, $\{1, 2, 3\}$ and $\{1, 2, 3, 4\}$ corresponds to the following conjunction of `cumulative` constraints

$$\mathtt{cumulative}\left(\left\langle\begin{array}{lll} \mathtt{origin}-o_1 & \mathtt{duration}-2 & \mathtt{height}-2, \\ \mathtt{origin}-o_2 & \mathtt{duration}-2 & \mathtt{height}-1 \end{array}\right\rangle, 2\right)$$

$$\mathtt{cumulative}\left(\left\langle\begin{array}{lll} \mathtt{origin}-o_1 & \mathtt{duration}-2 & \mathtt{height}-2, \\ \mathtt{origin}-o_2 & \mathtt{duration}-2 & \mathtt{height}-1, \\ \mathtt{origin}-o_3 & \mathtt{duration}-1 & \mathtt{height}-3 \end{array}\right\rangle, 3\right)$$

Even if the `cumulative` constraint could achieve arc-consistency, the previous conjunction of `cumulative` constraints would not detect the fact that there is no solution.

See also

common keyword: `cumulative` *(resource constraint)*.

used in graph description: `sum_ctr`.

Keywords

characteristic of a constraint: derived collection.

constraint type: scheduling constraint, resource constraint, temporal constraint.

modelling: zero-duration task.

Derived Collection

$$\texttt{col}\left(\begin{array}{l}\texttt{TIME\_POINTS}-\texttt{collection}\left(\begin{array}{l}\texttt{idp}-\texttt{int},\\\texttt{duration}-\texttt{dvar},\\\texttt{point}-\texttt{dvar}\end{array}\right),\\\left[\begin{array}{l}\texttt{item}\left(\begin{array}{l}\texttt{idp}-\texttt{TASKS.priority},\\\texttt{duration}-\texttt{TASKS.duration},\\\texttt{point}-\texttt{TASKS.origin}\end{array}\right),\\\texttt{item}\left(\begin{array}{l}\texttt{idp}-\texttt{TASKS.priority},\\\texttt{duration}-\texttt{TASKS.duration},\\\texttt{point}-\texttt{TASKS.end}\end{array}\right)\end{array}\right]\end{array}\right)$$

Arc input(s) TASKS

Arc generator $SELF \mapsto \texttt{collection}(\texttt{tasks})$

Arc arity 1

Arc constraint(s) $\texttt{tasks.origin} + \texttt{tasks.duration} = \texttt{tasks.end}$

Graph property(ies) $\mathbf{NARC} = |\texttt{TASKS}|$

For all items of PRIORITIES:

Arc input(s) TIME_POINTS TASKS

Arc generator $PRODUCT \mapsto \texttt{collection}(\texttt{time\_points}, \texttt{tasks})$

Arc arity 2

Arc constraint(s)
- $\texttt{time\_points.idp} = \texttt{PRIORITIES.id}$
- $\texttt{time\_points.idp} \geq \texttt{tasks.priority}$
- $\texttt{time\_points.duration} > 0$
- $\texttt{tasks.origin} \leq \texttt{time\_points.point}$
- $\texttt{time\_points.point} < \texttt{tasks.end}$

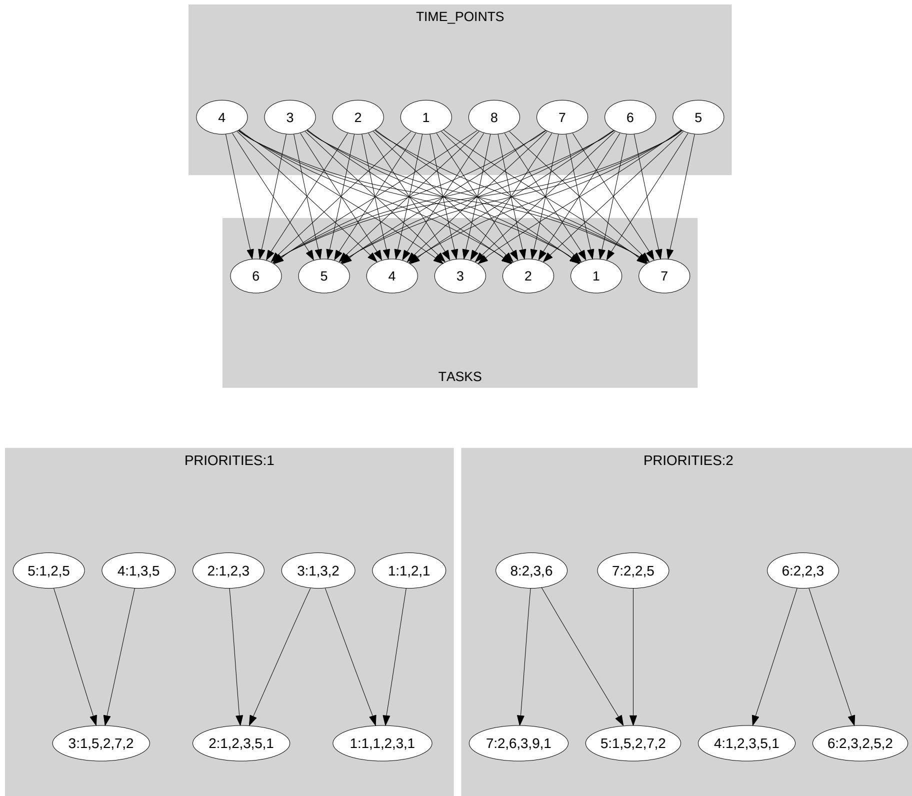
Graph class
- ACYCLIC
- BIPARTITE
- NO_LOOP

Sets

$$\mathsf{SUCC} \mapsto \left[\begin{array}{l}\texttt{source},\\\texttt{variables}-\texttt{col}\left(\begin{array}{l}\texttt{VARIABLES}-\texttt{collection}(\texttt{var}-\texttt{dvar}),\\ [\texttt{item}(\texttt{var}-\texttt{TASKS.height})]\end{array}\right)\end{array}\right]$$

Constraint(s) on sets $\texttt{sum\_ctr}(\texttt{variables}, \leq, \texttt{PRIORITIES.capacity})$

Graph model Within the context of the second graph constraint, part (A) of Figure 5.224 shows the initial graphs associated with priorities 1 and 2 of the **Example** slot. Part (B) of Figure 5.224 shows the corresponding final graphs associated with priorities 1 and 2. On the one hand, each source vertex of the final graph can be interpreted as a time point p. On the other hand the successors of a source vertex correspond to those tasks that both overlap that time point p and have a priority less than or equal to a given level. The cumulative_with_level_of_priority constraint holds since for each successor set $\mathcal{S}$ of the final graph the sum of the height of the tasks in $\mathcal{S}$ is less than or equal to the capacity associated with a given level of priority.



(A)

(B)

Figure 5.224: Initial and final graph of the `cumulative_with_level_of_priority` constraint

Signature

Since `TASKS` is the maximum number of vertices of the final graph of the first graph constraint we can rewrite $\mathbf{NARC} = |\mathtt{TASKS}|$ to $\mathbf{NARC} \geq |\mathtt{TASKS}|$. This leads to simplify $\underline{\overline{\mathbf{NARC}}}$ to $\overline{\mathbf{NARC}}$.