5.114 derangement

DESCRIPTION LINKS GRAPH

Origin

Derived from `cycle`.

Constraint

`derangement(NODES)`

Argument

NODES : collection(index−int, succ−dvar)

Restrictions

$|\mathtt{NODES}| > 1$
required(NODES, [index, succ])
$\mathtt{NODES.index} \geq 1$
$\mathtt{NODES.index} \leq |\mathtt{NODES}|$
distinct(NODES, index)
$\mathtt{NODES.succ} \geq 1$
$\mathtt{NODES.succ} \leq |\mathtt{NODES}|$

Purpose

Enforce to have a permutation with no cycle of length one. The permutation is depicted by the `succ` attribute of the `NODES` collection.

Example

$$\left(\left\langle\begin{array}{ll} \mathtt{index}-1 & \mathtt{succ}-2, \\ \mathtt{index}-2 & \mathtt{succ}-1, \\ \mathtt{index}-3 & \mathtt{succ}-5, \\ \mathtt{index}-4 & \mathtt{succ}-3, \\ \mathtt{index}-5 & \mathtt{succ}-4 \end{array}\right\rangle\right)$$

In the permutation of the example we have the following 2 cycles: $1 \rightarrow 2 \rightarrow 1$ and $3 \rightarrow 5 \rightarrow 4 \rightarrow 3$. Since these cycles have both a length strictly greater than one the corresponding `derangement` constraint holds.

Typical

$|\mathtt{NODES}| > 2$

Symmetries

- Items of `NODES` are permutable.
- Attributes of `NODES` are permutable w.r.t. permutation (`index`, `succ`) (*permutation applied to all items*).

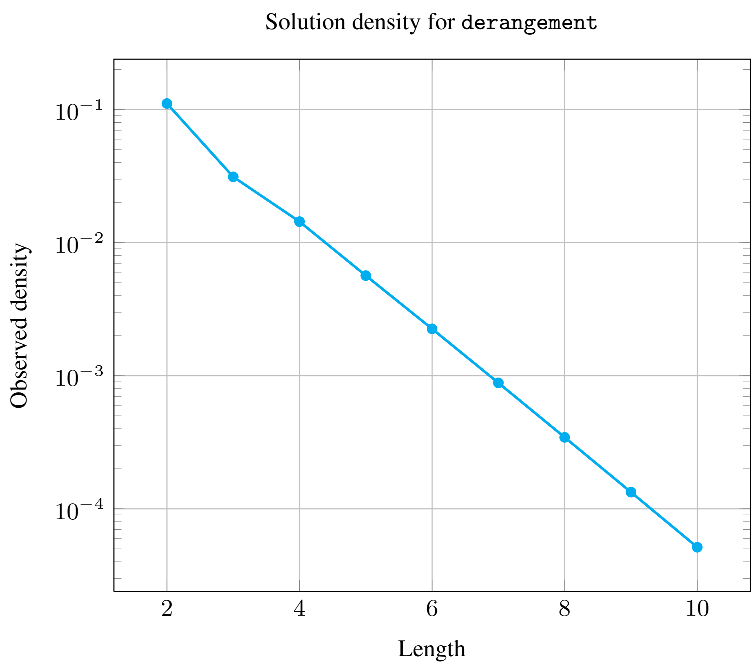
Remark

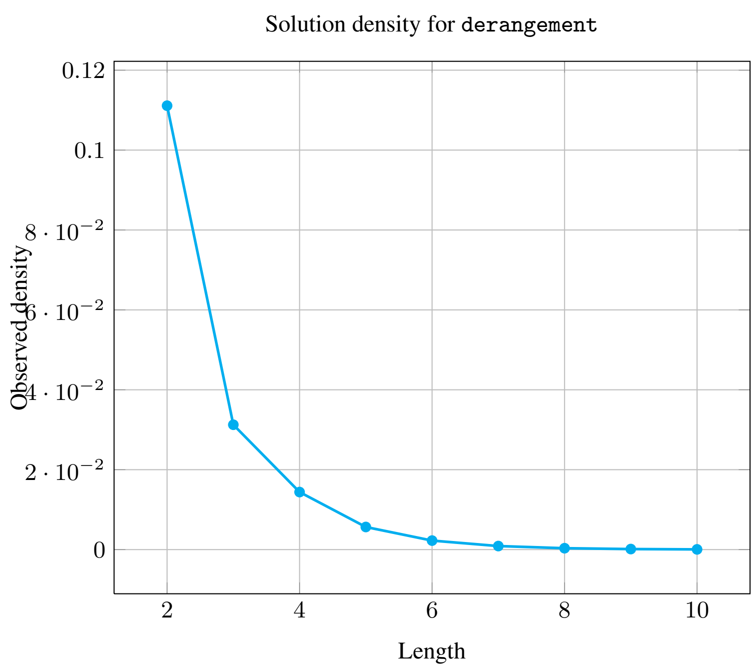
A special case of the `cycle` [41] constraint.

Counting

Length (n)	2	3	4	5	6	7	8	9	10
Solutions	1	2	9	44	265	1854	14833	133496	1334961

Number of solutions for `derangement`: domains $0..n$





See also **common keyword:** `alldifferent`, `cycle` *(permutation)*.

implied by: `symmetric_alldifferent`.

implies: `twin`.

implies (items to collection): `k_alldifferent`, `lex_alldifferent`.

Keywords

characteristic of a constraint: sort based reformulation.

combinatorial object: permutation.

constraint type: graph constraint.

filtering: arc-consistency, DFS-bottleneck.

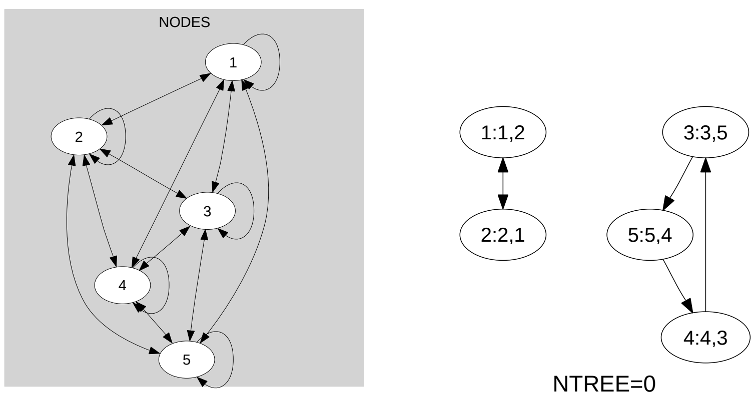
final graph structure: one_succ.

Cond. implications

`derangement`(NODES)
implies `permutation`(VARIABLES : NODES).

Arc input(s)	NODES
Arc generator	$CLIQUE \mapsto$ collection(nodes1, nodes2)
Arc arity	2
Arc constraint(s)	• nodes1.succ = nodes2.index • nodes1.succ $\neq$ nodes1.index
Graph property(ies)	**NTREE** = 0
Graph class	ONE_SUCC

Graph model Parts (A) and (B) of Figure 5.254 respectively show the initial and final graph associated with the **Example** slot. The derangement constraint holds since the final graph does not contain any vertex that does not belong to a circuit (i.e., **NTREE** = 0).



(A) **(B)**

Figure 5.254: Initial and final graph of the derangement constraint

In order to express the binary constraint that links two vertices of the NODES collection one has to make explicit the index value of the vertices. This is why the derangement constraint considers objects that have two attributes:

- One fixed attribute index that is the identifier of the vertex,
- One variable attribute succ that is the successor of the vertex.

Forbidding cycles of length one is achieved by the second condition of the arc constraint.

Signature Since 0 is the smallest possible value of **NTREE** we can rewrite the graph property **NTREE** = 0 to **NTREE** $\leq$ 0. This leads to simplify $\overline{\textbf{NTREE}}$ to $\underline{\textbf{NTREE}}$.