5.120 diffn_include

DESCRIPTION   **LINKS**   **GRAPH**

Origin   CHIP: option guillotine cut (include) of diffn.

Constraint   diffn_include(ORTHOTOPES, DIM)

Type   ORTHOTOPE : collection(ori−dvar, siz−dvar, end−dvar)

Arguments   ORTHOTOPES : collection(orth − ORTHOTOPE)
                     DIM   : int

Restrictions |ORTHOTOPE| > 0
                         require_at_least(2, ORTHOTOPE, [ori, siz, end])
                         ORTHOTOPE.siz ≥ 0
                         ORTHOTOPE.ori ≤ ORTHOTOPE.end
                         required(ORTHOTOPES, orth)
                         same_size(ORTHOTOPES, orth)
                         DIM > 0
                         DIM ≤ |ORTHOTOPE|
                         diffn(ORTHOTOPES)

Purpose

Extension of the generalised multi-dimensional non-overlapping diffn constraint. Holds if, for each pair of orthotopes (O_1, O_2) the following conditions hold:

- O_1 and O_2 do not overlap. Two orthotopes do not overlap if one of the orthotopes has zero size or if there exists at least one dimension where their projections do not overlap.

- Let P_1 and P_2 respectively denote the projections of O_1 and O_2 onto dimension DIM. If P_1 and P_2 overlap then, either P_1 is included in P_2, either P_2 is included in P_1.

Example

$$
\left\langle
\begin{array}{l}
\text{orth} - \langle \text{ori} - 8 \text{ siz} - 1 \text{ end} - 9, \text{ori} - 4 \text{ siz} - 1 \text{ end} - 5 \rangle, \\
\text{orth} - \left\langle \begin{array}{lll} \text{ori} - 9 & \text{siz} - 1 & \text{end} - 10, \\ \text{ori} - 4 & \text{siz} - 3 & \text{end} - 7 \end{array} \right\rangle, \\
\text{orth} - \langle \text{ori} - 6 \text{ siz} - 3 \text{ end} - 9, \text{ori} - 5 \text{ siz} - 2 \text{ end} - 7 \rangle, \\
\text{orth} - \langle \text{ori} - 1 \text{ siz} - 3 \text{ end} - 4, \text{ori} - 6 \text{ siz} - 1 \text{ end} - 7 \rangle, \\
\text{orth} - \langle \text{ori} - 4 \text{ siz} - 2 \text{ end} - 6, \text{ori} - 3 \text{ siz} - 4 \text{ end} - 7 \rangle, \\
\text{orth} - \left\langle \begin{array}{lll} \text{ori} - 6 & \text{siz} - 4 & \text{end} - 10, \\ \text{ori} - 1 & \text{siz} - 1 & \text{end} - 2 \end{array} \right\rangle, \\
\text{orth} - \left\langle \begin{array}{lll} \text{ori} - 10 & \text{siz} - 1 & \text{end} - 11, \\ \text{ori} - 1 & \text{siz} - 1 & \text{end} - 2 \end{array} \right\rangle, \\
\text{orth} - \left\langle \begin{array}{lll} \text{ori} - 6 & \text{siz} - 5 & \text{end} - 11, \\ \text{ori} - 2 & \text{siz} - 2 & \text{end} - 4 \end{array} \right\rangle, \\
\text{orth} - \langle \text{ori} - 6 \text{ siz} - 2 \text{ end} - 8, \text{ori} - 4 \text{ siz} - 1 \text{ end} - 5 \rangle, \\
\text{orth} - \langle \text{ori} - 1 \text{ siz} - 5 \text{ end} - 6, \text{ori} - 1 \text{ siz} - 2 \text{ end} - 3 \rangle, \\
\text{orth} - \langle \text{ori} - 1 \text{ siz} - 3 \text{ end} - 4, \text{ori} - 3 \text{ siz} - 2 \text{ end} - 5 \rangle, \\
\text{orth} - \langle \text{ori} - 1 \text{ siz} - 2 \text{ end} - 3, \text{ori} - 5 \text{ siz} - 1 \text{ end} - 6 \rangle
\end{array}
\right\rangle, 1
$$

Figure 5.270 represents the respective position of the twelve rectangles of the example. The coordinates of the leftmost lowest corner of each rectangle are stressed in bold. The `diffn_include` constraint holds since (1) the twelve rectangles do not overlap and since (2) when their projection onto dimension `DIM = 1` overlap one of the projections is included within the other one.
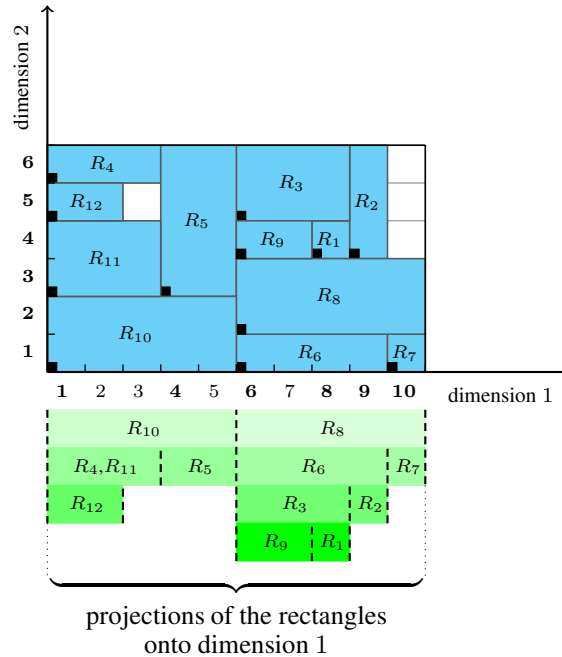

Figure 5.270: Illustration of the **Example** slot: twelve non-overlapping rectangles such that, for each pair of rectangles R_i, R_j ($1 \leq i < j \leq 12$), if the projections onto dimension 1 of rectangles R_i and R_j intersect then one of the projections is included within the other projection

Typical	$\|\text{ORTHOTOPE}\| > 1$
	$\text{ORTHOTOPE.siz} > 0$
	$\|\text{ORTHOTOPES}\| > 1$

| **Symmetries** | • Items of `ORTHOTOPES` are permutable. |
| | • One and the same constant can be added to the `ori` and `end` attributes of all items of `ORTHOTOPES.orth`. |

| **Arg. properties** | Contractible wrt. `ORTHOTOPES`. |

| **See also** | **common keyword:** diffn (*geometrical constraint,orthotope*), diffn_column (*geometrical constraint,orthotope,positioning constraint*). |

implied by: `diffn_column`.

used in graph description: `two_orth_column`.

Keywords   **constraint type:** decomposition.

geometry: geometrical constraint, positioning constraint, orthotope.

Arc input(s)	ORTHOTOPES
Arc generator	$CLIQUE(<) \mapsto$ collection(orthotopes1, orthotopes2)
Arc arity	2
Arc constraint(s)	two_orth_include(orthotopes1.orth, orthotopes2.orth, DIM)
Graph property(ies)	**NARC**$= |$ORTHOTOPES$| * (|$ORTHOTOPES$| - 1)/2$

Graph model
Since showing all items produces too big graphs, parts (A) and (B) of Figure 5.271 respectively show the initial and final graph associated with the first three items of the **Example** slot. Since we use the **NARC** graph property, the arcs of the final graph are stressed in bold.
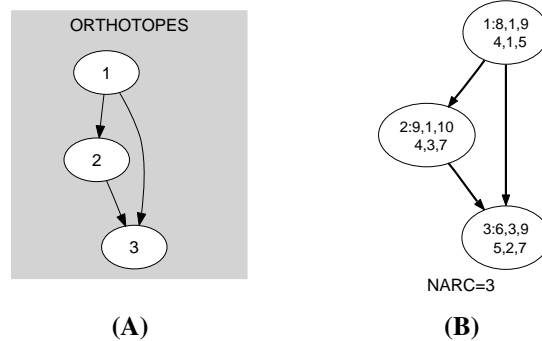

(A)   **(B)**

Figure 5.271: Initial and final graph of the diffn_include constraint