5.139 element

DESCRIPTION LINKS GRAPH AUTOMATON

Origin [418]

Constraint element(INDEX, TABLE, VALUE)

Synonyms nth, element_var, array.

Arguments

INDEX : dvar
TABLE : collection(value−dvar)
VALUE : dvar

Restrictions

INDEX ≥ 1
INDEX $\leq$ |TABLE|
|TABLE| > 0
required(TABLE, value)

Purpose VALUE is equal to the INDEXth item of TABLE, i.e. VALUE = TABLE[INDEX].

Example $(3, \langle 6, 9, 2, 9\rangle, 2)$

The element constraint holds since its third argument VALUE $=$ 2 is equal to the 3^{th} (INDEX $= 3$) item of the collection $\langle 6, 9, 2, 9\rangle$.

All solutions Figure 5.301 gives all solutions to the following non ground instance of the element constraint: $I \in [3, 6]$, $V \in [1, 9]$, element$(I, \langle 4, 8, 1, 0, 3, 3, 4, 3\rangle, V)$.

① $(3, \langle 4_1, 8_2, 1_3, 0_4, 3_4, 3_6, 7_7, 3_8\rangle, 1)$
② $(5, \langle 4_1, 8_2, 1_3, 0_4, 3_5, 3_6, 4_7, 3_8\rangle, 3)$
③ $(6, \langle 4_1, 8_2, 1_3, 0_4, 3_5, 3_6, 4_7, 3_8\rangle, 3)$

Figure 5.301: All solutions corresponding to the non ground example of the element constraint of the **All solutions** slot

Typical

|TABLE| > 1
range(TABLE.value) > 1

Symmetry All occurrences of two distinct values in TABLE.value or VALUE can be swapped; all occurrences of a value in TABLE.value or VALUE can be renamed to any unused value.

Arg. properties

- Functional dependency: VALUE determined by INDEX and TABLE.
- Suffix-extensible wrt. TABLE.

Usage

See **Usage** slot of `elem`.

Remark

In the original `element` constraint of **CHIP** the `index` attribute was not explicitly present in the table of values. It was implicitly defined as the position of a value in the previous table.

Within some systems (e.g., **Gecode**), the index of the first entry of the table TABLE corresponds to 0 rather than to 1.

When the first entry of the table TABLE corresponds to a value p that is different from 1 we can still use the `element` constraint. We use the reformulation $I = J - p + 1 \wedge$ `element`$(I, \texttt{TABLE}, V)$, where I and J are domain variables respectively ranging from 1 to $|\texttt{TABLE}|$ and from p to $p + |\texttt{TABLE}| - 1$.

The `element` constraint is called `nth` in **Choco** (http://choco.sourceforge.net/).
It is also sometimes called `element_var` when the second argument corresponds to a table of variables.

The `case` constraint [99] is a generalisation of the `element` constraint, where the table is replaced by a directed acyclic graph describing the set of solutions: there is a one to one correspondence between the solutions and the paths from the unique source of the dag to its leaves.

Systems

nth in **Choco**, element in **Gecode**, element in **JaCoP**, element in **MiniZinc**, element in **SICStus**.

See also

common keyword: `elem_from_to`, `element_greatereq`, `element_lesseq`, `element_matrix`, `element_product`, `element_sparse` *(array constraint)*, `elementn`, `elements_sparse`, `in_relation`, `stage_element`, `sum` *(data constraint)*.

generalisation: `cond_lex_cost` *(*`variable` *replaced by* `tuple` *of* `variables`*)*.

implied by: `elem`.

implies: `elem`.

related: `twin` *((pairs linked by an element with the same table))*.

system of constraints: `elements`.

uses in its reformulation: `cycle`, `elements_alldifferent`, `sort_permutation`, `tree_range`, `tree_resource`.

Keywords

characteristic of a constraint: core, automaton, automaton without counters, reified automaton constraint, derived collection.

constraint arguments: pure functional dependency.

constraint network structure: centered cyclic(2) constraint network(1).

constraint type: data constraint.

filtering: arc-consistency.

heuristics: labelling by increasing cost, regret based heuristics.

modelling: array constraint, table, functional dependency, variable indexing, variable subscript, disjunction, assignment to the same set of values, sequence dependent set-up.

modelling exercises: assignment to the same set of values, sequence dependent set-up, zebra puzzle.

puzzles: zebra puzzle.

Derived Collection

$\texttt{col}\left(\begin{array}{l}\texttt{ITEM}-\texttt{collection}(\texttt{index}-\texttt{dvar},\texttt{value}-\texttt{dvar}),\\ [\texttt{item}(\texttt{index}-\texttt{INDEX},\texttt{value}-\texttt{VALUE})]\end{array}\right)$

Arc input(s) ITEM TABLE

Arc generator $PRODUCT \mapsto \texttt{collection}(\texttt{item},\texttt{table})$

Arc arity 2

Arc constraint(s)
- $\texttt{item.index} = \texttt{table.key}$
- $\texttt{item.value} = \texttt{table.value}$

Graph property(ies) $\mathbf{NARC} = 1$

Graph model

The original `element` constraint with three arguments. We use the derived collection `ITEM` for putting together the `INDEX` and `VALUE` parameters of the `element` constraint. Within the arc constraint we use the implicit attribute `key` that associates to each item of a collection its position within the collection.

Parts (A) and (B) of Figure 5.302 respectively show the initial and final graph associated with the **Example** slot. Since we use the $\mathbf{NARC}$ graph property, the unique arc of the final graph is stressed in bold.

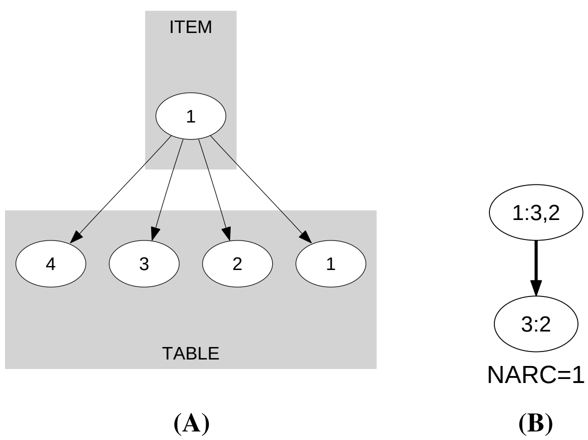


Figure 5.302: Initial and final graph of the `element` constraint

Signature

Because of the first condition of the arc constraint the final graph cannot have more than one arc. Therefore we can rewrite $\mathbf{NARC} = 1$ to $\mathbf{NARC} \geq 1$ and simplify $\overline{\mathbf{NARC}}$ to $\underline{\mathbf{NARC}}$.

Automaton

Figure 5.303 depicts the automaton associated with the `element` constraint. Let VALUE_i be the `value` attribute of item i of the TABLE collection. To each triple $(\text{INDEX}, \text{VALUE}, \text{VALUE}_i)$ corresponds a 0-1 signature variable S_i as well as the following signature constraint: $(\text{INDEX} = i \wedge \text{VALUE} = \text{VALUE}_i) \Leftrightarrow S_i$.

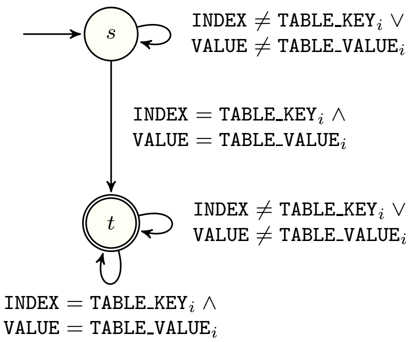


Figure 5.303: Automaton of the `element`(INDEX, TABLE, VALUE) constraint (once one finds the right index and value in the table, one switches from the initial state s to the accepting state t)

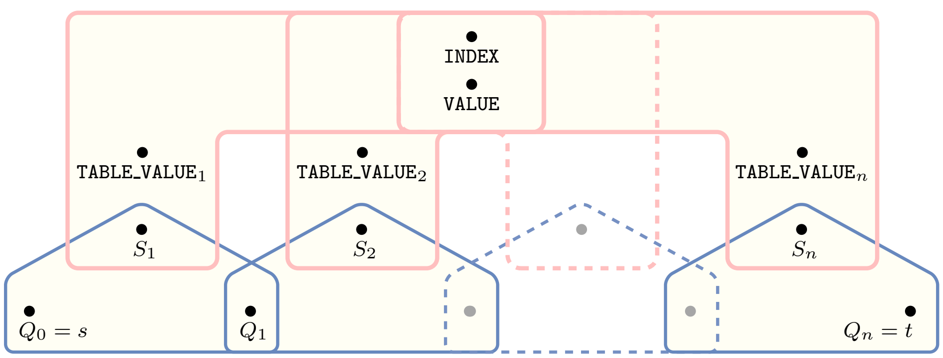


Figure 5.304: Hypergraph of the reformulation corresponding to the automaton of the `element` constraint

Quiz

EXERCISE 1 (checking whether a ground instance holds or not)[a]

A. Does the constraint `element`$(0, \langle 5, 1, 4, 8, 1\rangle, 5)$ hold?

B. Does the constraint `element`$(3, \langle 8, 2, 4, 3\rangle, 4)$ hold?

C. Does the constraint `element`$(5, \langle 0, 1, 2, 3, 4, 5\rangle, 5)$ hold?

[a] Hint: go back to the definition of `element`.

EXERCISE 2 (finding all solutions)[a]

Give all the solutions to the constraint:

$$\begin{cases} I \in [2,6], \\ V \in [0,5], \\ \texttt{element}(I, \langle 0,2,9,5,9,2,3,9\rangle, V). \end{cases}$$

[a]Hint: follow the order induced by the functional dependency between the arguments of `element`, enumerate solutions in lexicographic order.

EXERCISE 3 (finding all solutions)[a]

Give all the solutions to the constraint:

$$\begin{cases} I \in [2,3], \\ V_1 \in [5,5], V_2 \in [3,5], V_3 \in [0,3], \\ V \in [1,2], \\ \texttt{element}(I, \langle V_1, V_2, V_3\rangle, V). \end{cases}$$

[a]Hint: first find the feasible values of the first argument, then enumerate solutions in lexicographic order.

EXERCISE 4 (identifying infeasible values)[a]

Identify all variable-value pairs (V_i, val) $(0 \leq i \leq 3)$, such that the following constraint has no solution when value val is assigned to variable V_i:

$$\begin{cases} V_0 \in [2,3],\ \ V_1 \in [2,4], \\ V_2 \in [0,4],\ \ V_3 \in [3,5], \\ \texttt{element}(V_0, \langle V_1, 0, V_2, 6\rangle, V_3). \end{cases}$$

[a]Hint: first find the feasible values of the first argument, then filter the other variables.

EXERCISE 5 (variable-based degree of violation)[a]

Compute the variable-based degree of violation[b] of the following constraints:

A. $\texttt{element}(0, \langle 2,2,2,2\rangle, 2)$,

B. $\texttt{element}(3, \langle 3,1,5,2,7\rangle, 4)$,

C. $\texttt{element}(8, \langle 5,5,8,5,5,0,7\rangle, 2)$.

[a]Hint: take advantage of the functional dependency.

[b]Given a constraint for which all variables are fixed, the *variable-based degree of violation* is the minimum number of variables to assign differently in order to satisfy the constraint.

EXERCISE 6 (using entailment for counting)[a]

A. Given an `element`$(i, \langle t_1, t_2, \ldots, t_n\rangle, v)$ constraint where i, $t_1, t_2, \ldots, t_n$, v are variables, what is the minimum number of variables to fix in order to achieve entailment.[b] We assume that the constraint has at least one solution.

B. Exploit entailment in order to compute the number of solutions to the constraint $i \in [1, 3]$, $v_1 \in [0, 1]$, $v_2 \in [1, 9]$, $v_3 \in [3, 5]$, $v \in [2, 7]$, `element`$(i, \langle v_1, v_2, v_3\rangle, v)$.

[a] Hint: take advantage of the functional dependency, use a case analysis on the first argument.

[b] A constraint is *entailed* if and only if it is for sure satisfied even though some of its variables are not fixed.

EXERCISE 7 (modelling with an unconstrained index)[a]

What does the `element` constraint model when its first argument, the index, is unconstrained?

[a] Hint: how would one define the set of solutions of the third argument?

EXERCISE 8 (modelling an index starting at 0)[a]

Given a table t whose entries are indexed at $[0, n]$ model the requirement $v = t[i]$.

[a] Hint: make a shift.

EXERCISE 9 (modelling indirection)[a]

Given a table t whose entries vary between 1 and 9, model the requirement $v = t[t[i]]$ as one or several constraints. What is the implicit assumption we have on the entries of the table?

[a] Hint: use more than one constraint.

SOLUTION TO EXERCISE 1

A. *No, since value the first argument starts at index* 1.

B. *Yes, since the third entry of the table is equal to* 4.

C. *No, since the fifth entry is equal to* 4 *(and not to* 5*).*

SOLUTION TO EXERCISE 2

the three solutions

$I,\quad \langle 0,2,9,5,9,2,3,9\rangle,\quad V$

① $(\mathbf{2}, \langle 0_1, \mathbf{2_2}, 9_3, 5_4, 9_5, 2_6, 3_7, 9_8\rangle, \mathbf{2})$

② $(\mathbf{4}, \langle 0_1, 2_2, 9_3, \mathbf{5_4}, 9_5, 2_6, 3_7, 9_8\rangle, \mathbf{5})$

③ $(\mathbf{6}, \langle 0_1, 2_2, 9_3, 5_4, 9_5, \mathbf{2_6}, 3_7, 9_8\rangle, \mathbf{2})$

1. *The active entries of the table are located between index* 2 *and* 6*, as shown in bold by* $\langle 0_1, \mathbf{2_2}, \mathbf{9_3}, \mathbf{5_4}, \mathbf{9_5}, \mathbf{2_6}, 3_7, 9_8\rangle$.
2. *Among these entries we restrict ourselves to those entries for which the value is located in the domain of variable* V*, i.e. in interval* $[0, 5]$*. The remaining entries are shown in bold, i.e.* $\langle 0_1, \mathbf{2_2}, 9_3, \mathbf{5_4}, 9_5, \mathbf{2_6}, 3_7, 9_8\rangle$.
3. *This leads to the three solutions* $I = \mathbf{2}$ $V = \mathbf{2}$, $I = \mathbf{4}$ $V = \mathbf{5}$ *and* $I = \mathbf{6}$ $V = \mathbf{2}$.

SOLUTION TO EXERCISE 3

the six solutions

$I,\ \langle V_1, V_2, V_3\rangle,\ V$

① $(\mathbf{3}, \langle 5_1, 3_2, \mathbf{1_3}\rangle, \mathbf{1})$

② $(\mathbf{3}, \langle 5_1, 3_2, \mathbf{2_3}\rangle, \mathbf{2})$

③ $(\mathbf{3}, \langle 5_1, 4_2, \mathbf{1_3}\rangle, \mathbf{1})$

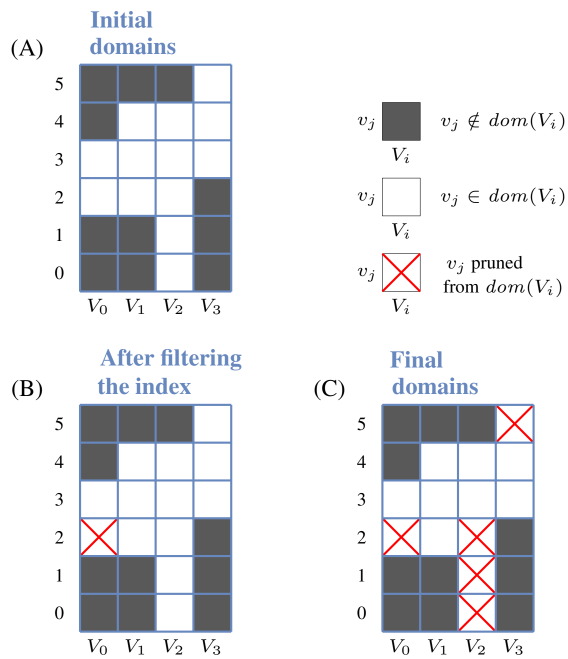
④ $(\mathbf{3}, \langle 5_1, 4_2, \mathbf{2_3}\rangle, \mathbf{2})$

⑤ $(\mathbf{3}, \langle 5_1, 5_2, \mathbf{1_3}\rangle, \mathbf{1})$

⑥ $(\mathbf{3}, \langle 5_1, 5_2, \mathbf{2_3}\rangle, \mathbf{2})$

1. *Since the domain of* V_2 *which is located at the second entry of the table does not intersect the domain of* V *(the third argument), the index variable* I *(the first argument) can not be assigned value* 2*, and is therefore fixed to* 3.
2. *Since* I *is fixed to* 3 *we have that* $V = V_3$*. Consequently* V *and* V_3 *are assigned a same value that belongs to the intersection of their respective domains, i.e.* $[0, 3] \cap [1, 2] = [1, 2]$.

SOLUTION TO EXERCISE 4



1. *In part* (A) *we give the initial domains of the index variable* (V_0), *of the first and third entries of the table* (V_1, V_2), *and of the third argument of the* `element` *constraint* (V_3).
2. *In part* (B) *we prune the index variable* V_0. *On the one hand, it can not be assigned value* 2 *since the second entry of the table is set to* 0, *and* 0 *does not belong to the domain of* V_3, *see* ×. *On the other hand, it can be assigned value* 3 *since* $dom(V_2) \cap dom(V_3) \neq \emptyset$.
3. *Finally in part* (C) *we remove those values that contradict the fact that* $V_2 = V_3$, *see* ×.

SOLUTION TO EXERCISE 5

A. *The degree of violation is equal to* 1 *since we only need to change the index from* 0 *(because* 0 *is not an allowed value for the index) to any integer value in* $[1, 4]$.

$$\texttt{element}(\overset{1}{0}, \langle 2, 2, 2, 2\rangle, 2)$$

B. *The degree of violation is equal to* 1 *since we only need to change the third entry of the table to* 4 *(or to switch the third argument from* 4 *to* 5*).*

$$\texttt{element}(3, \langle 3, 1, \overset{4}{5}, 2, 7\rangle, 4)$$

C. *The degree of violation is equal to* 2 *since we need to change both the index (the table has only* 7 *entries) and the third argument (value* 2 *does not occur in the table). Rather than changing the third argument, we may change an entry of the table (e.g., if we set the index to* 3 *we set the third entry of the table to* 2*).*

$$\texttt{element}(\overset{1}{8}, \langle 5, 5, 8, 5, 5, 0, 7\rangle, \overset{5}{2})$$

SOLUTION TO EXERCISE 6

A. We need to fix 3 variables in the following way:

- **(i)** The first argument, the index i, is fixed to a value α $(1 \leq \alpha \leq n)$ such that $dom(t_\alpha) \cap dom(v) \neq \emptyset$.
- **(ii)** We fix the third argument v to a value β in $dom(t_\alpha) \cap dom(v)$.
- **(iii)** We fix t_α to β.

B. We have 90 solutions depending on whether $i = 1$, $i = 2$, $i = 3$ (and $v = v_i$):

- **(i)** $|dom(v) \cap dom(v_1)| \cdot |dom(v_2)| \cdot |dom(v_3)| = 0 \cdot 9 \cdot 3 = 0$,
- **(ii)** $|dom(v) \cap dom(v_2)| \cdot |dom(v_1)| \cdot |dom(v_3)| = 6 \cdot 2 \cdot 3 = 36$,
- **(iii)** $|dom(v) \cap dom(v_3)| \cdot |dom(v_1)| \cdot |dom(v_2)| = 3 \cdot 2 \cdot 9 = 54$.

SOLUTION TO EXERCISE 7

Given a table t *of* n *entries* $t[1], t[2], \ldots, t[n]$, `element` *models a disjunction stating that the third argument* v *is equal to one of the values that can be assigned to one of the variables of the table, i.e.* $v = t[1] \vee v = t[2] \vee \cdots \vee v = t[n]$.

SOLUTION TO EXERCISE 8

The requirement $v = t[i]$ *can be modelled as the conjunction of the two constraints* $j = i + 1$, element$(j, \langle t[0], t[1], \ldots, t[n]\rangle, v)$.

SOLUTION TO EXERCISE 9

The requirement $v = t[t[i]]$ *can be modelled as the conjunction of two* element *constraints sharing the same table, namely:*

element$(i, \langle t[1], t[2], \ldots, t[9]\rangle, j)$ $\leftarrow$ *inner indirection* $t[t[i]]$

element$(j, \langle t[1], t[2], \ldots, t[9]\rangle, v)$ $\leftarrow$ *outer indirection* $t[t[i]]$

The second element *constraint assumes that* j *corresponds to a valid index of the table, i.e. a value between* 1 *and* 9.