5.147   elements_alldifferent

Origin   Derived from elements and alldifferent.

Constraint   elements_alldifferent(ITEMS, TABLE)

Synonyms   elements_alldiff, elements_alldistinct.

Arguments   ITEMS : collection(index−dvar, value−dvar)
                      TABLE : collection(index−int, value−dvar)

Restrictions   required(ITEMS, [index, value])
                      ITEMS.index ≥ 1
                      ITEMS.index $\leq$ |TABLE|
                      |ITEMS| $=$ |TABLE|
                      required(TABLE, [index, value])
                      TABLE.index ≥ 1
                      TABLE.index $\leq$ |TABLE|
                      distinct(TABLE, index)

Purpose   All the items of the ITEMS collection should be equal to one of the entries of the table TABLE and all the variables ITEMS.index should take distinct values.

Example
$$\left(\left\langle \begin{array}{ll} index-2 & value-9, \\ index-1 & value-6, \\ index-4 & value-9, \\ index-3 & value-2 \end{array} \right\rangle, \left\langle \begin{array}{ll} index-1 & value-6, \\ index-2 & value-9, \\ index-3 & value-2, \\ index-4 & value-9 \end{array} \right\rangle \right)$$

The elements_alldifferent constraint holds since, as depicted by Figure 5.321, there is a one to one correspondence between the items of the ITEMS collection and the items of the TABLE collection.
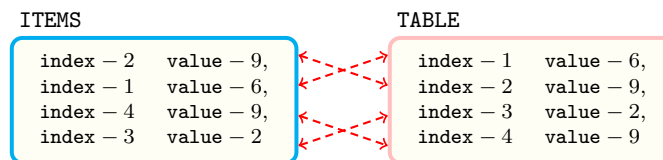

Figure 5.321: Illustration of the one to one correspondence between the items of ITEMS and the items of TABLE

Typical	$|\texttt{ITEMS}| > 1$
	$\texttt{range}(\texttt{ITEMS.value}) > 1$
	$|\texttt{TABLE}| > 1$
	$\texttt{range}(\texttt{TABLE.value}) > 1$

Symmetries

- Arguments are permutable w.r.t. permutation (ITEMS, TABLE).
- Items of ITEMS are permutable.
- Items of TABLE are permutable.
- All occurrences of two distinct values in ITEMS.value or TABLE.value can be swapped; all occurrences of a value in ITEMS.value or TABLE.value can be renamed to any unused value.

Arg. properties

Functional dependency: ITEMS.value determined by ITEMS.index and TABLE.

Usage

Used for replacing by a single elements_alldifferent constraint an alldifferent and a set of element constraints having the following structure:

- The union of the index variables of the element constraints is equal to the set of variables of the alldifferent constraint.
- All the element constraints share exactly the same table.

For instance, the constraint given in the **Example** slot is equivalent to the conjunction of the following set of constraints:

$$\texttt{alldifferent}(\langle \texttt{var} - 2, \texttt{var} - 1, \texttt{var} - 4, \texttt{var} - 3\rangle)$$

$$
\texttt{element}
\left(
\begin{array}{l}
\langle\ \texttt{index} - 2 \quad \texttt{value} - 9\ \rangle, \\
\quad\ \ \left\langle
\begin{array}{l}
\texttt{index} - 1 \quad \texttt{value} - 6, \\
\texttt{index} - 2 \quad \texttt{value} - 9, \\
\texttt{index} - 3 \quad \texttt{value} - 2, \\
\texttt{index} - 4 \quad \texttt{value} - 9
\end{array}
\right\rangle
\end{array}
\right)
$$

$$
\texttt{element}
\left(
\begin{array}{l}
\langle\ \texttt{index} - 1 \quad \texttt{value} - 6\ \rangle, \\
\quad\ \ \left\langle
\begin{array}{l}
\texttt{index} - 1 \quad \texttt{value} - 6, \\
\texttt{index} - 2 \quad \texttt{value} - 9, \\
\texttt{index} - 3 \quad \texttt{value} - 2, \\
\texttt{index} - 4 \quad \texttt{value} - 9
\end{array}
\right\rangle
\end{array}
\right)
$$

$$
\texttt{element}
\left(
\begin{array}{l}
\langle\ \texttt{index} - 3 \quad \texttt{value} - 2\ \rangle, \\
\quad\ \ \left\langle
\begin{array}{l}
\texttt{index} - 1 \quad \texttt{value} - 6, \\
\texttt{index} - 2 \quad \texttt{value} - 9, \\
\texttt{index} - 3 \quad \texttt{value} - 2, \\
\texttt{index} - 4 \quad \texttt{value} - 9
\end{array}
\right\rangle
\end{array}
\right)
$$

$$
\texttt{element}
\left(
\begin{array}{l}
\langle\ \texttt{index} - 4 \quad \texttt{value} - 9\ \rangle, \\
\quad\ \ \left\langle
\begin{array}{l}
\texttt{index} - 1 \quad \texttt{value} - 6, \\
\texttt{index} - 2 \quad \texttt{value} - 9, \\
\texttt{index} - 3 \quad \texttt{value} - 2, \\
\texttt{index} - 4 \quad \texttt{value} - 9
\end{array}
\right\rangle
\end{array}
\right)
$$

As a practical example of utilisation of the `elements_alldifferent` constraint we show how to model the link between a permutation consisting of a single cycle and its expanded form. For instance, to the permutation $3, 6, 5, 2, 4, 1$ corresponds the sequence 3 5 4 2 6 1. Let us note $S_1, S_2, S_3, S_4, S_5, S_6$ the permutation and $V_1 V_2 V_3 V_4 V_5 V_6$ its expanded form (see Figure 5.322).

The constraint:

$$
\texttt{elements\_alldifferent}
\left(
\left\langle
\begin{array}{ll}
\texttt{index} - V_1 & \texttt{value} - V_2, \\
\texttt{index} - V_2 & \texttt{value} - V_3, \\
\texttt{index} - V_3 & \texttt{value} - V_4, \\
\texttt{index} - V_4 & \texttt{value} - V_5, \\
\texttt{index} - V_5 & \texttt{value} - V_6, \\
\texttt{index} - V_6 & \texttt{value} - V_1
\end{array}
\right\rangle,
\left\langle
\begin{array}{ll}
\texttt{index} - 1 & \texttt{value} - S_1, \\
\texttt{index} - 2 & \texttt{value} - S_2, \\
\texttt{index} - 3 & \texttt{value} - S_3, \\
\texttt{index} - 4 & \texttt{value} - S_4, \\
\texttt{index} - 5 & \texttt{value} - S_5, \\
\texttt{index} - 6 & \texttt{value} - S_6
\end{array}
\right\rangle
\right)
$$

models the fact that $S_1, S_2, S_3, S_4, S_5, S_6$ corresponds to a permutation with a single cycle. It also expresses the link between the variables $S_1, S_2, S_3, S_4, S_5, S_6$ and $V_1, V_2, V_3, V_4, V_5, V_6$.
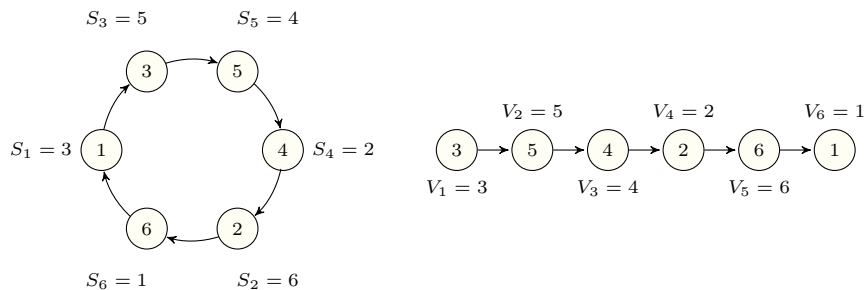

Figure 5.322: Two representations of a permutation containing a single cycle

Reformulation

The `elements_alldifferent`($\langle \texttt{index} - I_1 \texttt{ value} - V_1, \texttt{index} - I_2 \texttt{ value} - V_2, \ldots, \texttt{index} - I_{|\texttt{ITEMS}|} \texttt{ value} - V_{|\texttt{ITEMS}|} \rangle$, TABLE) constraint can be expressed in term of a conjunction of $|\texttt{ITEMS}|$ `elem` constraints and of one `alldifferent` constraint of the form:

> `elem`($\langle \texttt{index} - I_1 \texttt{ value} - V_1 \rangle$, TABLE),
> `elem`($\langle \texttt{index} - I_2 \texttt{ value} - V_2 \rangle$, TABLE),
> ...
> `elem`($\langle \texttt{index} - I_{|\texttt{ITEMS}|} \texttt{ value} - V_{|\texttt{ITEMS}|} \rangle$, TABLE),
> `alldifferent`($\langle I_1, I_2, \ldots, I_{|\texttt{ITEMS}|} \rangle$).

See also **implies:** elements, indexed_sum.

used in reformulation: alldifferent, elem, element.

Keywords **characteristic of a constraint:** disequality.

combinatorial object: permutation.

constraint type: data constraint.

modelling: array constraint, table, functional dependency.

Cond. implications elements_alldifferent(ITEMS, TABLE)

with TABLE.value ≥ 0

implies bin_packing_capa(TABLE, ITEMS).

Arc input(s)	ITEMS TABLE
Arc generator	$PRODUCT \mapsto$ collection(items, table)
Arc arity	2
Arc constraint(s)	• items.index $=$ table.index • items.value $=$ table.value
Graph property(ies)	**NVERTEX**$= |$ITEMS$| + |$TABLE$|$

Graph model

The fact that all variables ITEMS.index are pairwise different is derived from the conjunctions of the following facts:

- From the graph property **NVERTEX** $= |$ITEMS$| + |$TABLE$|$ it follows that all vertices of the initial graph belong also to the final graph,

- A vertex v belongs to the final graph if there is at least one constraint involving v that holds,

- From the first condition items.index $=$ table.index of the arc constraint, and from the restriction distinct(TABLE.index) it follows: for all vertices v generated from the collection ITEMS at most one constraint involving v holds.

Parts (A) and (B) of Figure 5.323 respectively show the initial and final graph associated with the **Example** slot. Since we use the **NVERTEX** graph property, the vertices of the final graph are stressed in bold.
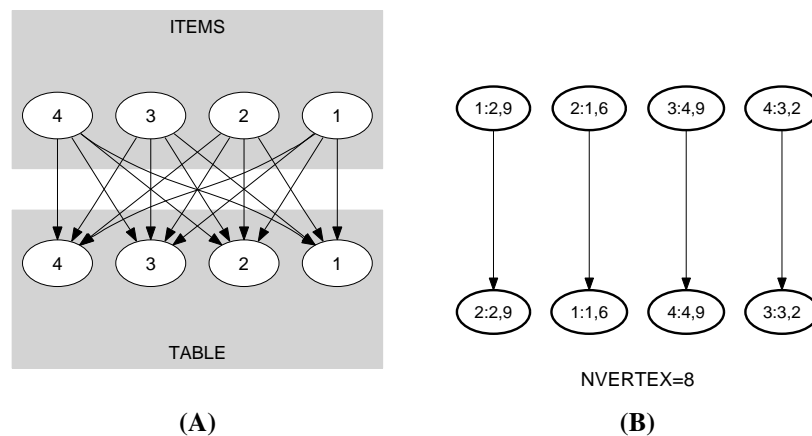

(A)   (B)

Figure 5.323: Initial and final graph of the elements_alldifferent constraint

Signature

Since the final graph cannot have more than $|$ITEMS$| + |$TABLE$|$ vertices one can simplify **NVERTEX** to **NVERTEX**.