5.155 exactly

DESCRIPTION LINKS GRAPH AUTOMATON

Origin Derived from atleast and atmost.

Constraint exactly(N, VARIABLES, VALUE)

Synonym count.

Arguments

N	:	int
VARIABLES	:	collection(var−dvar)
VALUE	:	int

Restrictions

$N \geq 0$
$N \leq |\text{VARIABLES}|$
required(VARIABLES, var)

Purpose Exactly N variables of the VARIABLES collection are assigned value VALUE.

Example $(2, \langle 4, 2, 4, 5\rangle, 4)$

The exactly constraint holds since exactly $N = 2$ variables of the $\text{VARIABLES} = \langle 4, 2, 4, 5\rangle$ collection are assigned value $\text{VALUE} = 4$.

Typical

$N > 0$
$N < |\text{VARIABLES}|$
$|\text{VARIABLES}| > 1$

Symmetries

- Items of VARIABLES are permutable.
- An occurrence of a value of VARIABLES.var that is different from VALUE can be replaced by any other value that is also different from VALUE.

Arg. properties

- Functional dependency: N determined by VARIABLES and VALUE.
- Aggregate: N(+), VARIABLES(union), VALUE(id).

Systems occurence in **Choco**, count in **Gecode**, exactly in **Gecode**, count in **JaCoP**, exactly in **MiniZinc**, count in **SICStus**.

See also **generalisation:** among (constant *replaced by* variable *and* value *replaced by* list *of* values).

implies: atleast (= N *replaced by* $\geq$ N), atmost (= N *replaced by* $\leq$ N).

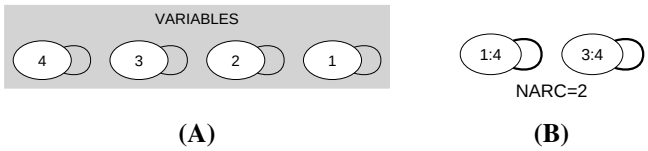
Keywords

characteristic of a constraint: automaton, automaton with counters.
constraint arguments: reverse of a constraint, pure functional dependency.
constraint network structure: alpha-acyclic constraint network(2).
constraint type: value constraint, counting constraint.
filtering: glue matrix, arc-consistency.
modelling: functional dependency.

Arc input(s)	VARIABLES
Arc generator	$SELF \mapsto$ collection(variables)
Arc arity	1
Arc constraint(s)	variables.var = VALUE
Graph property(ies)	NARC= N

Graph model Since each arc constraint involves only one vertex (VALUE is fixed), we employ the $SELF$ arc generator in order to produce a graph with a single loop on each vertex.

Parts (A) and (B) of Figure 5.332 respectively show the initial and final graph associated with the **Example** slot. Since we use the NARC graph property, the loops of the final graph are stressed in bold. The exactly constraint holds since exactly two variables are assigned value 4.

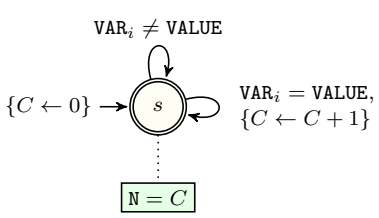


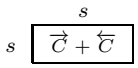
(A) **(B)**

Figure 5.332: Initial and final graph of the exactly constraint

Automaton

Figure 5.333 depicts the automaton associated with the `exactly` constraint. To each variable $\texttt{VAR}_i$ of the collection `VARIABLES` corresponds a 0-1 signature variable S_i. The following signature constraint links $\texttt{VAR}_i$ and S_i: $\texttt{VAR}_i = \texttt{VALUE} \Leftrightarrow S_i$.





Glue matrix where $\overrightarrow{C}$ and $\overleftarrow{C}$ resp. represent the counter value C at the end of a prefix and at the end of the corresponding reverse suffix that partitions the sequence `VARIABLES`.

Figure 5.333: Automaton (with one counter) of the `exactly` constraint and its glue matrix

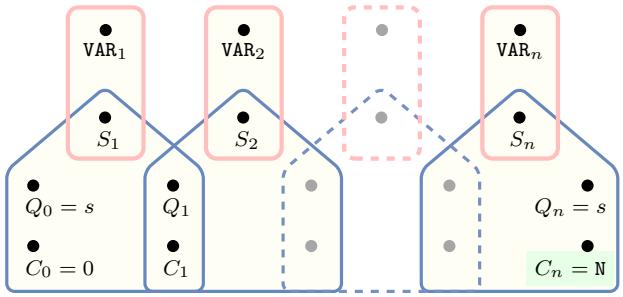


Figure 5.334: Hypergraph of the reformulation corresponding to the automaton (with one counter) of the `exactly` constraint: since all states variables $Q_0, Q_1, \ldots, Q_n$ are fixed to the unique state s of the automaton, the transitions constraints share only the counter variable C and the constraint network is Berge-acyclic