5.173 group_skip_isolated_item

DESCRIPTION LINKS GRAPH AUTOMATON

Origin

Derived from group.

Constraint

group_skip_isolated_item(NGROUP, MIN_SIZE, MAX_SIZE, NVAL, VARIABLES, VALUES)

Arguments

NGROUP	:	dvar
MIN_SIZE	:	dvar
MAX_SIZE	:	dvar
NVAL	:	dvar
VARIABLES	:	collection(var−dvar)
VALUES	:	collection(val−int)

Restrictions

NGROUP ≥ 0
$3 *$ NGROUP $\leq$ |VARIABLES| $+ 1$
MIN_SIZE ≥ 0
MIN_SIZE $\neq 1$
MAX_SIZE $\geq$ MIN_SIZE
NVAL $\geq$ MAX_SIZE
NVAL $\geq$ NGROUP
NVAL $\leq$ |VARIABLES|
required(VARIABLES, var)
required(VALUES, val)
distinct(VALUES, val)

Purpose

Let n be the number of variables of the collection VARIABLES. Let $X_i, X_{i+1}, \ldots, X_j$ $(1 \leq i < j \leq n)$ be consecutive variables of the collection of variables VARIABLES such that the following conditions apply:

- All variables $X_i, \ldots, X_j$ take their value in the set of values VALUES,
- $i = 1$ or X_{i-1} does not take a value in VALUES,
- $j = n$ or X_{j+1} does not take a value in VALUES.

We call such a set of variables a *group*. The constraint group_skip_isolated_item is true if all the following conditions hold:

- There are exactly NGROUP groups of variables,
- The number of variables of the smallest group is MIN_SIZE,
- The number of variables of the largest group is MAX_SIZE,
- The number of variables that take their value in the set of values VALUES is equal to NVAL.

Example

$$(1, 2, 2, 3, \langle 2, 8, 1, 7, 4, 5, 1, 1, 1\rangle, \langle 0, 2, 4, 6, 8\rangle)$$

Given the fact that groups are formed by even values in $\{0, 2, 4, 6, 8\}$ (i.e., values expressed by the `VALUES` collection), and the fact that isolated even values are ignored, the `group_skip_isolated_item` constraint holds since:

- Its first argument, `NGROUP`, is set to value 1 since the sequence 2 8 1 7 4 5 1 1 1 contains only one group of even values involving more than one even value (i.e., group 2 8).
- Its second and third arguments, `MIN_SIZE` and `MAX_SIZE`, are both set to 2 since the only group of even values with more than one even value involves two values (i.e., group 2 8).
- The fourth argument, `NVAL`, is fixed to 2 since it corresponds to the total number of even values belonging to groups involving more than one even value (i.e., value 4 is discarded since it is an isolated even value of the sequence 2 8 1 7 4 5 1 1 1).

Typical

$\texttt{NGROUP} > 0$
$\texttt{MIN\_SIZE} > 0$
$\texttt{NVAL} > \texttt{MAX\_SIZE}$
$\texttt{NVAL} > \texttt{NGROUP}$
$\texttt{NVAL} < |\texttt{VARIABLES}|$
$|\texttt{VARIABLES}| > 1$
$\text{range}(\texttt{VARIABLES.var}) > 1$
$|\texttt{VALUES}| > 0$
$|\texttt{VARIABLES}| > |\texttt{VALUES}|$

Symmetries

- Items of `VARIABLES` can be reversed.
- Items of `VALUES` are permutable.
- An occurrence of a value of `VARIABLES.var` that belongs to `VALUES.val` (resp. does not belong to `VALUES.val`) can be replaced by any other value in `VALUES.val` (resp. not in `VALUES.val`).

Arg. properties

- Functional dependency: `NGROUP` determined by `VARIABLES` and `VALUES`.
- Functional dependency: `MIN_SIZE` determined by `VARIABLES` and `VALUES`.
- Functional dependency: `MAX_SIZE` determined by `VARIABLES` and `VALUES`.
- Functional dependency: `NVAL` determined by `VARIABLES` and `VALUES`.

Usage

This constraint is useful in order to specify rules about how rest days should be allocated to a person during a period of n consecutive days. In this case `VALUES` are the codes for the rest days (perhaps a single value) and `VARIABLES` corresponds to the amount of work done during n consecutive days. We can then express a rule like: in a month one should have at least 4 periods of at least 2 rest days (isolated rest days are not counted as rest periods).

Remark

The following invariant imposes a limit on the maximum number of groups wrt the minimum size of a group and the total number of variables: $\texttt{NGROUP} \cdot (\max(\texttt{MIN\_SIZE}, 2) + 1) \leq |\texttt{VARIABLES}| + 1$.

See also

common keyword: `change_continuity`, `group`, `stretch_path` *(timetabling constraint,sequence)*.

used in graph description: `in`.

Keywords

characteristic of a constraint: automaton, automaton with counters, automaton with same input symbol.

combinatorial object: sequence.

constraint arguments: reverse of a constraint.

constraint network structure: alpha-acyclic constraint network(2), alpha-acyclic constraint network(3).

constraint type: timetabling constraint.

filtering: glue matrix.

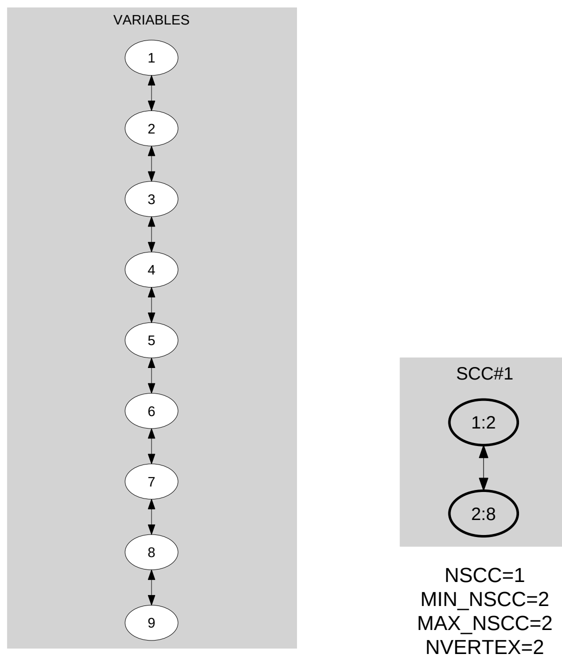
final graph structure: strongly connected component.

modelling: functional dependency.

Arc input(s)	VARIABLES
Arc generator	$CHAIN \mapsto$ collection(variables1, variables2)
Arc arity	2
Arc constraint(s)	• in(variables1.var, VALUES) • in(variables2.var, VALUES)
Graph property(ies)	• **NSCC**= NGROUP • **MIN_NSCC**= MIN_SIZE • **MAX_NSCC**= MAX_SIZE • **NVERTEX**= NVAL

Graph model We use the $CHAIN$ arc generator in order to produce the initial graph. In the context of the **Example** slot, this creates the graph depicted in part (A) of Figure 5.384. We use $CHAIN$ together with the arc constraint variables1.var $\in$ VALUES$\wedge$variables2.var $\in$ VALUES in order to skip the isolated variables that take a value in VALUES that we do not want to count as a group. This is why, on the example, value 4 is not counted as a group. Part (B) of Figure 5.384 shows the final graph associated with the **Example** slot. The group_skip_isolated_item constraint of the **Example** slot holds since:

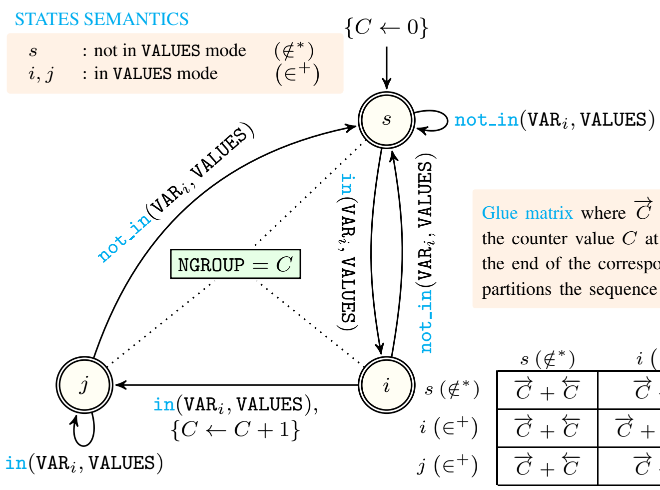
- The final graph contains one strongly connected component. Therefore the number of groups is equal to one.
- The unique strongly connected component of the final graph contains two vertices. Therefore MIN_SIZE and MAX_SIZE are both equal to 2.
- The number of vertices of the final graph is equal to two. Therefore NVAL is equal to 2.



(A) **(B)**

Figure 5.384: Initial and final graph of the `group_skip_isolated_item` constraint

Automaton Figures 5.385, 5.387, 5.389 and 5.391 depict the different automata associated with the `group_skip_isolated_item` constraint. For the automata that respectively compute `NGROUP`, `MIN_SIZE`, `MAX_SIZE` and `NVAL` we have a 0-1 signature variable $\mathtt{S}_i$ for each variable $\mathtt{VAR}_i$ of the collection `VARIABLES`. The following signature constraint links $\mathtt{VAR}_i$ and $\mathtt{S}_i$: $\mathtt{VAR}_i \in \mathtt{VALUES} \Leftrightarrow \mathtt{S}_i$.



Glue matrix where $\overrightarrow{C}$ and $\overleftarrow{C}$ resp. represent the counter value C at the end of a prefix and at the end of the corresponding reverse suffix that partitions the sequence `VARIABLES`.

	$s\ (\notin^*)$	$i\ (\in^+)$	$j\ (\in^+)$
$s\ (\notin^*)$	$\overrightarrow{C}+\overleftarrow{C}$	$\overrightarrow{C}+\overleftarrow{C}$	$\overrightarrow{C}+\overleftarrow{C}$
$i\ (\in^+)$	$\overrightarrow{C}+\overleftarrow{C}$	$\overrightarrow{C}+1+\overleftarrow{C}$	$\overrightarrow{C}+\overleftarrow{C}$
$j\ (\in^+)$	$\overrightarrow{C}+\overleftarrow{C}$	$\overrightarrow{C}+\overleftarrow{C}$	$\overrightarrow{C}-1+\overleftarrow{C}$

Figure 5.385: Automaton for the `NGROUP` argument of the `group_skip_isolated_item` constraint and its glue matrix

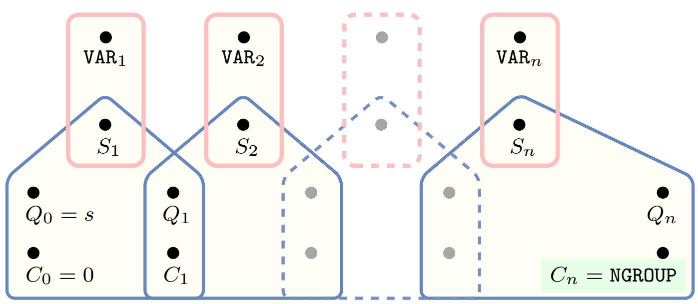
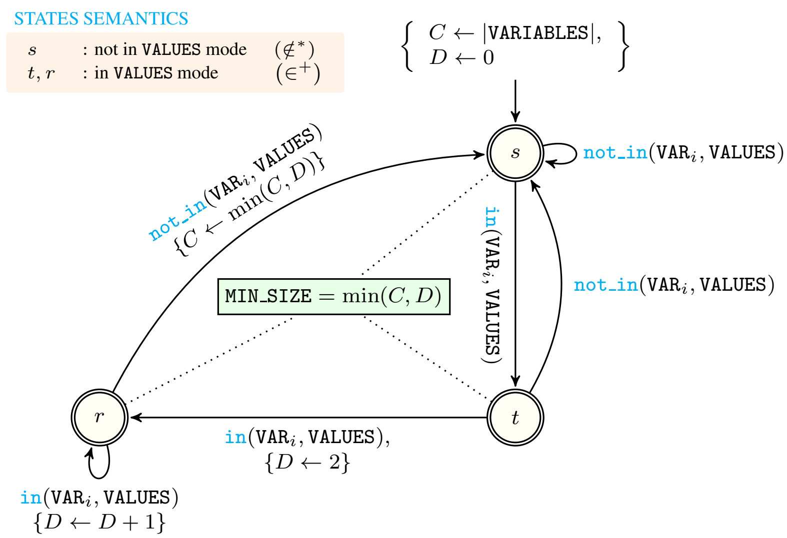


Figure 5.386: Hypergraph of the reformulation corresponding to the automaton (with one counter) of the `NGROUP` argument of the `group_skip_isolated_item` constraint (since all states of the automaton are accepting there is no restriction on the last variable Q_n)



Glue matrix where $\overrightarrow{C}$, $\overrightarrow{D}$ and $\overleftarrow{C}$, $\overleftarrow{D}$ resp. represent the counters values C, D at the end of a prefix and at the end of the corresponding reverse suffix that partitions the sequence VARIABLES.

	s ($\notin^*$)	t ($\in^+$)	r ($\in^+$)
s ($\notin^*$)	$\min(\overrightarrow{C}, \overrightarrow{D} + \overleftarrow{D}, \overleftarrow{C})$	$\min(\overrightarrow{C}, \overrightarrow{D} + \overleftarrow{D}, \overleftarrow{C})$	$\min(\overrightarrow{C}, \overleftarrow{D}, \overleftarrow{C})$
t ($\in^+$)	$\min(\overrightarrow{C}, \overrightarrow{D} + \overleftarrow{D}, \overleftarrow{C})$	2	$\min(\overrightarrow{C}, \overleftarrow{D} + 1, \overleftarrow{C})$
r ($\in^+$)	$\min(\overrightarrow{C}, \overrightarrow{D}, \overleftarrow{C})$	$\min(\overrightarrow{C}, \overrightarrow{D} + 1, \overleftarrow{C})$	$\min(\overrightarrow{C}, \overrightarrow{D} + \overleftarrow{D}, \overleftarrow{C})$

Figure 5.387: Automaton for the MIN_SIZE argument of the group_skip_isolated_item constraint and its glue matrix

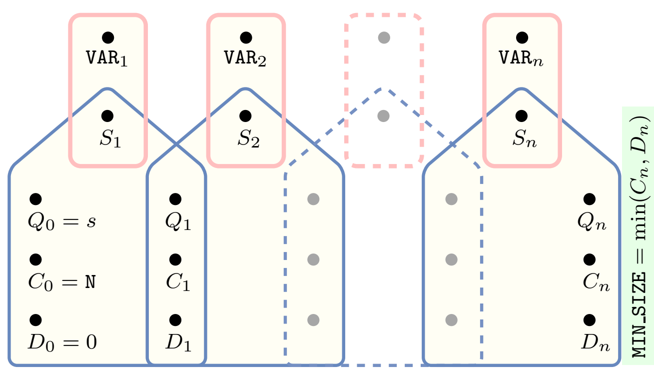
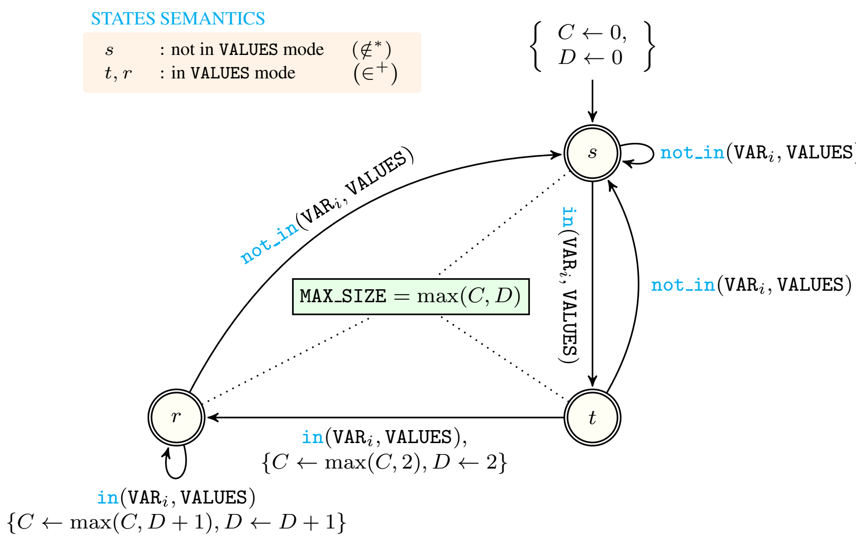


Figure 5.388: Hypergraph of the reformulation corresponding to the automaton (with two counters) of the MIN_SIZE argument of the group_skip_isolated_item constraint where N stands for |VARIABLES| (since all states of the automaton are accepting there is no restriction on the last variable Q_n)



Glue matrix where $\overrightarrow{C}$, $\overrightarrow{D}$ and $\overleftarrow{C}$, $\overleftarrow{D}$ resp. represent the counters values C, D at the end of a prefix and at the end of the corresponding reverse suffix that partitions the sequence VARIABLES.

	s ($\notin^*$)	t ($\in^+$)	r ($\in^+$)
s ($\notin^*$)	$\max(\overrightarrow{C}, \overleftarrow{C})$	$\max(\overrightarrow{C}, \overleftarrow{C})$	$\max(\overrightarrow{C}, \overleftarrow{C})$
t ($\in^+$)	$\max(\overrightarrow{C}, \overleftarrow{C})$	$\max(\overrightarrow{C}, 2, \overleftarrow{C})$	$\max(\overrightarrow{C}, \overleftarrow{D}+1, \overleftarrow{C})$
r ($\in^+$)	$\max(\overrightarrow{C}, \overleftarrow{C})$	$\max(\overrightarrow{C}, \overrightarrow{D}+1, \overleftarrow{C})$	$\max(\overrightarrow{C}, \overrightarrow{D}+\overleftarrow{D}, \overleftarrow{C})$

Figure 5.389: Automaton for the `MAX_SIZE` argument of the `group_skip_isolated_item` constraint and its glue matrix

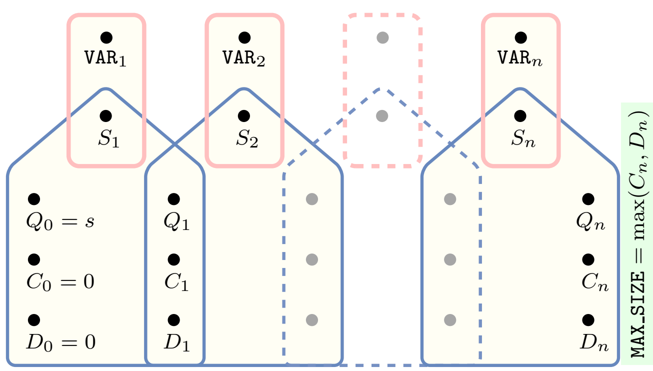


Figure 5.390: Hypergraph of the reformulation corresponding to the automaton (with two counters) of the `MAX_SIZE` argument of the `group_skip_isolated_item` constraint (since all states of the automaton are accepting there is no restriction on the last variable Q_n)

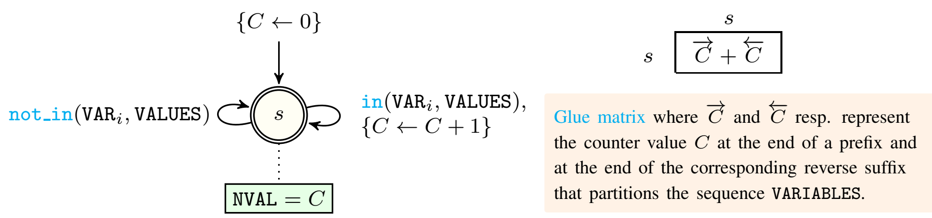


Figure 5.391: Automaton for the NVAL argument of the group_skip_isolated_item constraint and its glue matrix

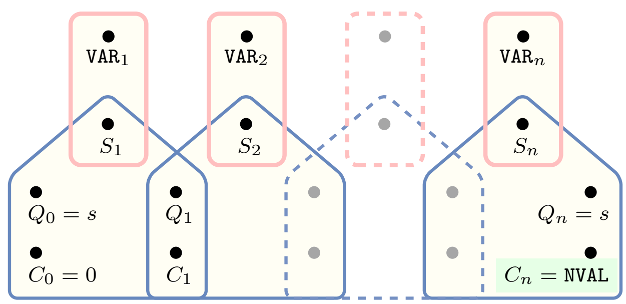


Figure 5.392: Hypergraph of the reformulation corresponding to the automaton (with one counter) of the NVAL argument of the group_skip_isolated_item constraint