## 5.188 increasing_nvalue_chain

DESCRIPTION LINKS GRAPH AUTOMATON

**Origin** Derived from increasing_nvalue.

**Constraint** increasing_nvalue_chain(NVAL, VARIABLES)

**Arguments**

NVAL : dvar
VARIABLES : collection(b−dvar, var−dvar)

**Restrictions**

$\texttt{NVAL} \geq \texttt{min}(1, |\texttt{VARIABLES}|)$
$\texttt{NVAL} \leq |\texttt{VARIABLES}|$
required(VARIABLES, [b, var])
$\texttt{VARIABLES.b} \geq 0$
$\texttt{VARIABLES.b} \leq 1$

**Purpose**

For each consecutive pair of items $\texttt{VARIABLES}[i], \texttt{VARIABLES}[i+1]$ $(1 \leq i < |\texttt{VARIABLES}|)$ of the VARIABLES collection at least one of the following conditions hold:

1. $\texttt{VARIABLES}[i+1].\texttt{b} = 0$,
2. $\texttt{VARIABLES}[i].\texttt{var} \leq \texttt{VARIABLES}[i+1].\texttt{var}$.

In addition, NVAL is equal to number of pairs of variables $\texttt{VARIABLES}[i], \texttt{VARIABLES}[i+1]$ $(1 \leq i < |\texttt{VARIABLES}|)$ plus one, which verify at least one of the following conditions:

1. $\texttt{VARIABLES}[i+1].\texttt{b} = 0$,
2. $\texttt{VARIABLES}[i].\texttt{var} < \texttt{VARIABLES}[i+1].\texttt{var}$.

Note that $\texttt{VARIABLES}[1].\texttt{b}$ is not referenced at all in the previous definition (i.e., its value does not influence at all the values assigned to the other variables).

**Example**

$$\left(6, \left\langle \begin{array}{ll} \texttt{b}-0 & \texttt{var}-2, \\ \texttt{b}-1 & \texttt{var}-4, \\ \texttt{b}-1 & \texttt{var}-4, \\ \texttt{b}-1 & \texttt{var}-4, \\ \texttt{b}-0 & \texttt{var}-4, \\ \texttt{b}-1 & \texttt{var}-8, \\ \texttt{b}-0 & \texttt{var}-1, \\ \texttt{b}-0 & \texttt{var}-7, \\ \texttt{b}-1 & \texttt{var}-7 \end{array} \right\rangle \right)$$

The increasing_nvalue_chain constraint holds since:

1. The condition $\texttt{VARIABLES}[i+1].\texttt{b} = 0 \vee \texttt{VARIABLES}[i].\texttt{var} \leq \texttt{VARIABLES}[i+1].\texttt{var}$ holds for every pair of adjacent items of the VARIABLES collection:

- For the pair $(\texttt{VARIABLES}[1].\texttt{var}, \texttt{VARIABLES}[2].\texttt{var})$ we have $\texttt{VARIABLES}[1].\texttt{var} \leq \texttt{VARIABLES}[2].\texttt{var}$ $(2 \leq 4)$.
- For the pair $(\texttt{VARIABLES}[2].\texttt{var}, \texttt{VARIABLES}[3].\texttt{var})$ we have $\texttt{VARIABLES}[2].\texttt{var} \leq \texttt{VARIABLES}[3].\texttt{var}$ $(4 \leq 4)$.
- For the pair $(\texttt{VARIABLES}[3].\texttt{var}, \texttt{VARIABLES}[4].\texttt{var})$ we have $\texttt{VARIABLES}[3].\texttt{var} \leq \texttt{VARIABLES}[4].\texttt{var}$ $(4 \leq 4)$.
- For the pair $(\texttt{VARIABLES}[4].\texttt{var}, \texttt{VARIABLES}[5].\texttt{var})$ we have $\texttt{VARIABLES}[5].\texttt{b} = 0$.
- For the pair $(\texttt{VARIABLES}[5].\texttt{var}, \texttt{VARIABLES}[6].\texttt{var})$ we have $\texttt{VARIABLES}[5].\texttt{var} \leq \texttt{VARIABLES}[6].\texttt{var}$ $(4 \leq 8)$.
- For the pair $(\texttt{VARIABLES}[6].\texttt{var}, \texttt{VARIABLES}[7].\texttt{var})$ we have $\texttt{VARIABLES}[7].\texttt{b} = 0$.
- For the pair $(\texttt{VARIABLES}[7].\texttt{var}, \texttt{VARIABLES}[8].\texttt{var})$ we have $\texttt{VARIABLES}[8].\texttt{b} = 0$.
- For the pair $(\texttt{VARIABLES}[8].\texttt{var}, \texttt{VARIABLES}[9].\texttt{var})$ we have $\texttt{VARIABLES}[8].\texttt{var} \leq \texttt{VARIABLES}[9].\texttt{var}$ $(7 \leq 7)$.

2. NVAL is equal to number of pairs of variables $\texttt{VARIABLES}[i], \texttt{VARIABLES}[i+1]$ $(1 \leq i < |\texttt{VARIABLES}|)$ plus one which verify at least $\texttt{VARIABLES}[i+1].\texttt{b} = 0 \vee \texttt{VARIABLES}[i].\texttt{var} < \texttt{VARIABLES}[i+1].\texttt{var}$. Beside the *plus one*, the following five pairs contribute for 1 in NVAL:

   - For the pair $(\texttt{VARIABLES}[1].\texttt{var}, \texttt{VARIABLES}[2].\texttt{var})$ we have $\texttt{VARIABLES}[1].\texttt{var} \leq \texttt{VARIABLES}[2].\texttt{var}$ $(2 < 4)$.
   - For the pair $(\texttt{VARIABLES}[4].\texttt{var}, \texttt{VARIABLES}[5].\texttt{var})$ we have $\texttt{VARIABLES}[5].\texttt{b} = 0$.
   - For the pair $(\texttt{VARIABLES}[5].\texttt{var}, \texttt{VARIABLES}[6].\texttt{var})$ we have $\texttt{VARIABLES}[5].\texttt{var} \leq \texttt{VARIABLES}[6].\texttt{var}$ $(4 < 8)$.
   - For the pair $(\texttt{VARIABLES}[6].\texttt{var}, \texttt{VARIABLES}[7].\texttt{var})$ we have $\texttt{VARIABLES}[7].\texttt{b} = 0$.
   - For the pair $(\texttt{VARIABLES}[7].\texttt{var}, \texttt{VARIABLES}[8].\texttt{var})$ we have $\texttt{VARIABLES}[8].\texttt{b} = 0$.

**Typical**

$|\texttt{VARIABLES}| > 1$
$\texttt{range}(\texttt{VARIABLES.b}) > 1$
$\texttt{range}(\texttt{VARIABLES.var}) > 1$

**See also**

**related:** `increasing_nvalue`, `nvalue`, `ordered_nvector`.

**Keywords**

**constraint type:** counting constraint, order constraint.

**modelling:** number of distinct values.

| | |
|---|---|
| **Arc input(s)** | VARIABLES |
| **Arc generator** | $PATH \mapsto$ `collection`(variables1, variables2) |
| **Arc arity** | 2 |
| **Arc constraint(s)** | variables2.b $= 0 \lor$ variables1.var $\leq$ variables2.var |
| **Graph property(ies)** | $\mathbf{NARC} = \lvert$VARIABLES$\rvert - 1$ |

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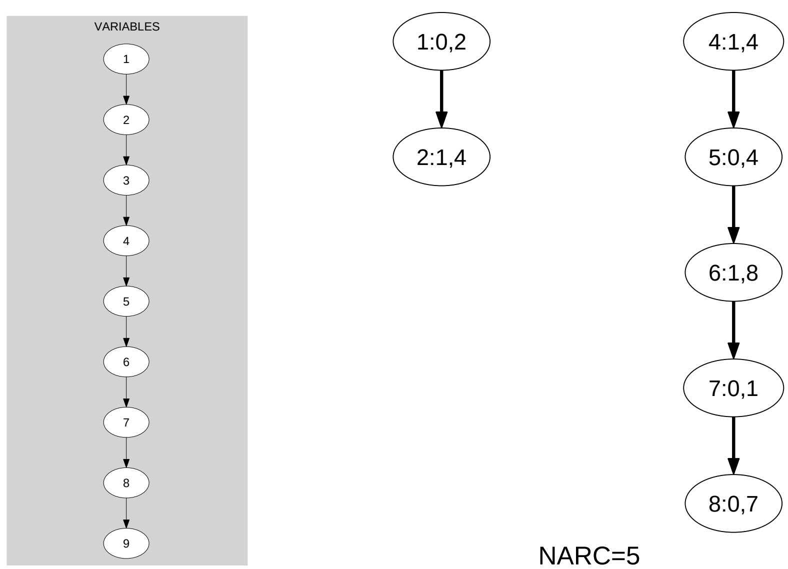
**Graph model** Parts (A) and (B) of Figure 5.419 respectively show the initial and final graph associated with the second graph constraint of the **Example** slot. Since we use the $\mathbf{NARC}$ graph property the arcs of the final graph are stressed in bold.




**(A)** **(B)**

Figure 5.419: Initial and final graph of the `increasing_nvalue_chain` constraint

**Automaton**

Without loss of generality, assume that the collection VARIABLES contains at least one variable (i.e., $|\texttt{VARIABLES}| \geq 1$). Let $l$, $m$, $n$, $min$ and $max$ respectively denote the minimum and maximum possible value of variable NVAL, the number of items of the collection VARIABLES, the smallest value that can be assigned to VARIABLES$[i]$.var $(1 \leq i \leq n)$, and the largest value that can be assigned to VARIABLES$[i]$.var $(1 \leq i \leq n)$. Let $s = max - min + 1$ denote the total number of potential values. Clearly, the maximum value of NVAL cannot exceed the quantity $d = \min(m, n)$. The states of the automaton that only accepts solutions of the increasing_nvalue_chain constraint can be defined in the following way:

- We have an initial state labelled by $s_{00}$.
- We have $d \cdot s$ states labelled by $s_{ij}$ $(1 \leq i \leq d, 1 \leq j \leq s)$.

Terminal states depend on the possible values of variable NVAL and correspond to those states $s_{ij}$ such that $i$ is a possible value for variable NVAL. Note that we assume no further restriction on the domain of NVAL (otherwise the set of accepting states needs to be reduced in order to reflect the current set of possible values of NVAL).

Transitions of the automaton are labelled by a pair of values $(\alpha, \beta)$ and correspond to a condition of the form VARIABLES$[i]$.b $= \alpha \wedge$ VARIABLES$[i]$.var $= \beta$, $(1 \leq i \leq n)$. Characters $*$ and $+$ respectively represent all values in $\{0, 1\}$ and all values in $\{min, min + 1, \ldots, max\}$. Four classes of transitions are respectively defined in the following way:

1. There is a transition, labelled by the pair $(*, min + j - 1)$, from the initial state $s_{00}$ to the state $s_{1j}$ $(1 \leq j \leq s)$. We use the $*$ character since VARIABLES$[1]$.b is not use at all in the definition of the increasing_nvalue_chain constraint.
2. There is a loop, labelled by the pair $(1, min + j - 1)$ for every state $s_{ij}$ $(1 \leq i \leq d, 1 \leq j \leq s)$.
3. $\forall i \in [1, d-1], \forall j \in [1, s], \forall k \in [j + 1, s]$ there is a transition labelled by the pair $(1, min + k - 1)$ from $s_{ij}$ to $s_{i+1k}$.
4. $\forall i \in [1, d-1], \forall j \in [1, s]$ there is a transition labelled by the pair $(0, +)$ from $s_{ij}$ to $s_{i+1\,1}$.

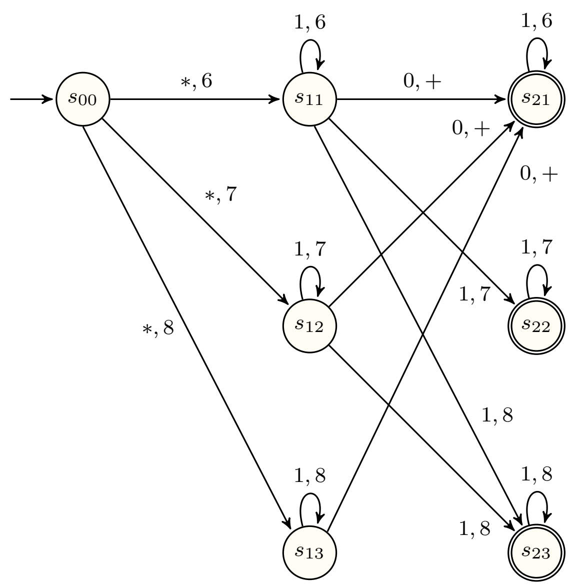



Figure 5.420: Automaton of the `increasing_nvalue_chain` constraint under the hypothesis that all variables are assigned a value in $\{6, 7, 8\}$ and that `NVAL` is equal to $2$. The character $*$ on a transition corresponds to a $0$ or to a $1$ and the $+$ corresponds to a 6, 7 or $8$.