5.193 inflexion

DESCRIPTION LINKS AUTOMATON

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Constraint `inflexion`(N, VARIABLES)

Arguments

N : dvar
VARIABLES : collection(var−dvar)

Restrictions

$\mathtt{N} \geq 0$
$\mathtt{N} \leq \mathtt{max}(0, |\mathtt{VARIABLES}| - 2)$
required(VARIABLES, var)

Purpose

N is equal to the number of times that the following conjunctions of constraints hold:

- X_i CTR $X_{i+1} \wedge X_i \neq X_{i+1}$,
- $X_{i+1} = X_{i+2} \wedge \cdots \wedge X_{j-2} = X_{j-1}$,
- $X_{j-1} \neq X_j \wedge X_{j-1}$ ¬CTR X_j.

where X_k is the k^{th} item of the VARIABLES collection and $1 \leq i, i+2 \leq j, j \leq n$ and CTR is $<$ or $>$.

Example

$(3, \langle 1, 1, 4, 8, 8, 2, 7, 1\rangle)$
$(0, \langle 1, 1, 4, 4, 6, 6, 7, 9\rangle)$
$(7, \langle 1, 0, 2, 0, 7, 2, 7, 1, 2\rangle)$

The first `inflexion` constraint holds since the sequence 1 1 4 8 8 2 7 1 contains three inflexions peaks that respectively correspond to values 8, 2 and 7.

All solutions

Figure 5.429 gives all solutions to the following non ground instance of the `inflexion` constraint: $\mathtt{N} \in \{0, 2\}$, $V_1 = 2$, $V_2 \in [2, 3]$, $V_3 \in [1, 2]$, $V_4 \in [1, 2]$, $V_5 = 3$, `inflexion`$(\mathtt{N}, \langle V_1, V_2, V_3, V_4, V_5\rangle)$.

Typical

$\mathtt{N} > 0$
$|\mathtt{VARIABLES}| > 2$
range(VARIABLES.var) > 1

Symmetries

- Items of VARIABLES can be reversed.
- One and the same constant can be added to the var attribute of all items of VARIABLES.

Arg. properties

Functional dependency: N determined by VARIABLES.

Usage

Useful for constraining the number of *inflexions* of a sequence of domain variables.

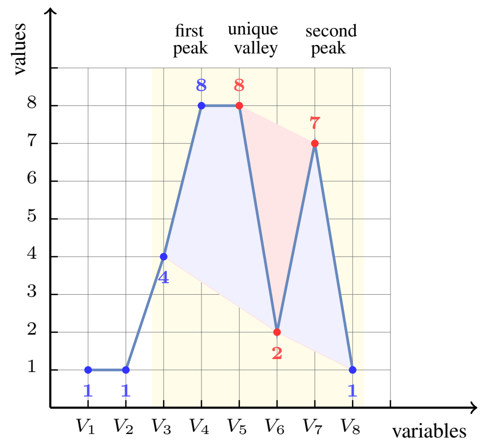


Figure 5.428: Illustration of the first example of the **Example** slot: a sequence of eight variables V_1, V_2, V_3, V_4, V_5, V_6, V_7, V_8 respectively fixed to values $1, 1, 4, 8, 8, 2, 7, 1$ and its three inflexions in red, two peaks and one valley ($\mathtt{N} = 3$)

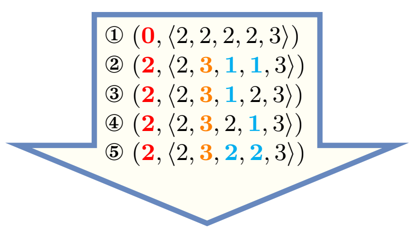


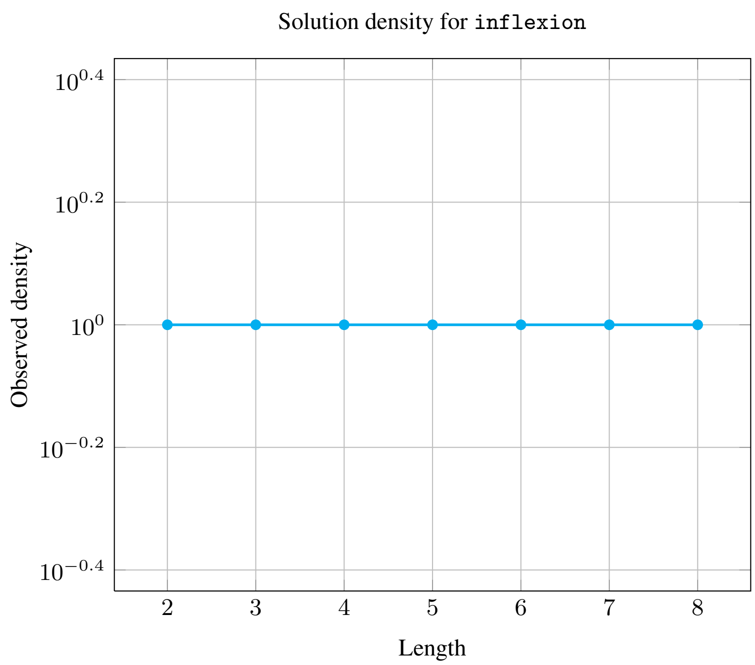
Figure 5.429: All solutions corresponding to the non ground example of the `inflexion` constraint of the **All solutions** slot where each inflexion (i.e. peak or valley) is coloured in orange or cyan

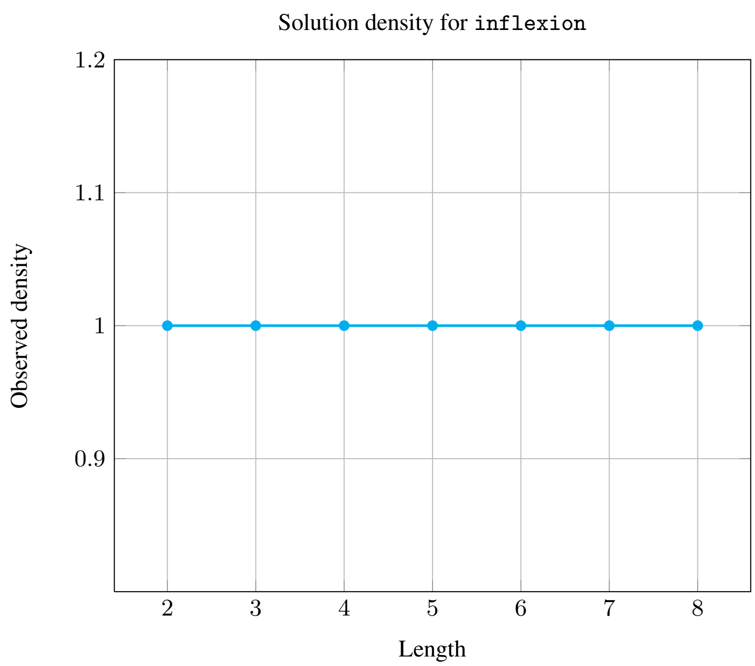
Remark Since the arity of the arc constraint is not fixed, the `inflexion` constraint cannot be currently described with the graph-based representation. However, this would not hold anymore if we were introducing a slot that specifies how to merge adjacent vertices of the final graph.

Counting

Length (n)	2	3	4	5	6	7	8
Solutions	9	64	625	7776	117649	2097152	43046721

Number of solutions for `inflexion`: domains $0..n$

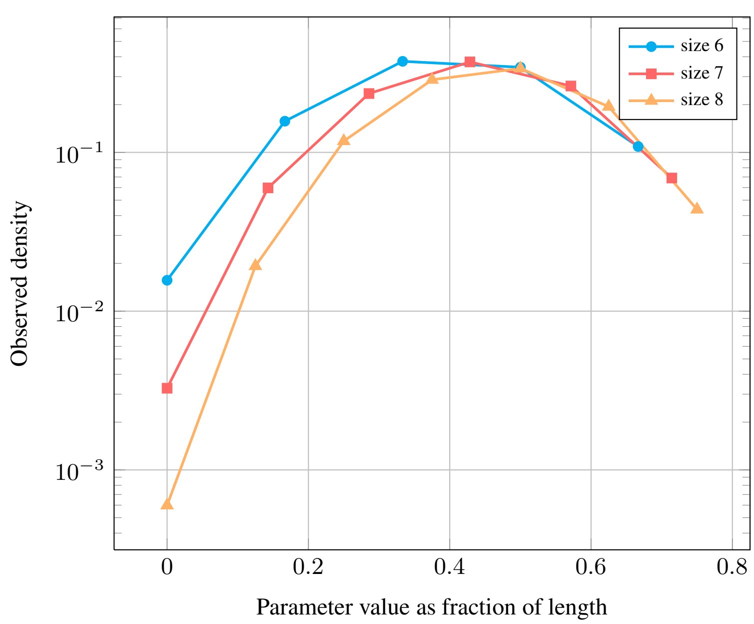

Solution density for inflexion
Observed density
10^0.4
10^0.2
10^0
10^-0.2
10^-0.4
2
3
4
5
6
7
8
Length

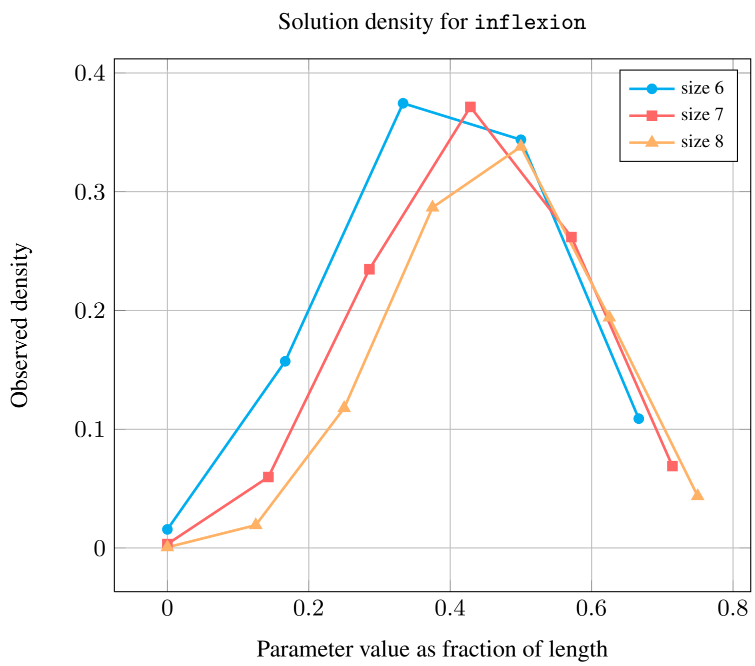

Solution density for inflexion
Observed density
1.2
1.1
1
0.9
2
3
4
5
6
7
8
Length

Length (n)		2	3	4	5	6	7	8
Total		9	64	625	7776	117649	2097152	43046721
Parameter value	0	9	36	135	498	1841	6856	25731
	1	-	28	320	2588	18494	125284	828120
	2	-	-	170	3348	44058	492320	5069970
	3	-	-	-	1342	40446	778936	12341184
	4	-	-	-	-	12810	549152	14547186
	5	-	-	-	-	-	144604	8354520
	6	-	-	-	-	-	-	1880010

Solution count for `inflexion`: domains $0..n$

Solution density for `inflexion`





See also **common keyword:** `global_contiguity`, `min_dist_between_inflexion`, `peak`, `valley` *(sequence)*.

Keywords **characteristic of a constraint:** automaton, automaton with counters, automaton with same input symbol.

combinatorial object: sequence.

constraint arguments: reverse of a constraint, pure functional dependency.

constraint network structure: sliding cyclic(1) constraint network(2).

filtering: glue matrix.

modelling: functional dependency.

Cond. implications

- `inflexion`(N, VARIABLES)
  with N > 0
  implies `atleast_nvalue`(NVAL, VARIABLES)
  when NVAL $= 2$.
- `inflexion`(N, VARIABLES)
  with `valley`(VARIABLES.var) $= 0$
  implies `peak`(N, VARIABLES).
- `inflexion`(N, VARIABLES)
  with `peak`(VARIABLES.var) $= 0$
  implies `valley`(N, VARIABLES).

Automaton Figure 5.430 depicts the automaton associated with the `inflexion` constraint. To each pair of consecutive variables $(\mathtt{VAR}_i, \mathtt{VAR}_{i+1})$ of the collection `VARIABLES` corresponds a signature variable S_i. The following signature constraint links $\mathtt{VAR}_i$, $\mathtt{VAR}_{i+1}$ and S_i: $(\mathtt{VAR}_i < \mathtt{VAR}_{i+1} \Leftrightarrow S_i = 0) \wedge (\mathtt{VAR}_i = \mathtt{VAR}_{i+1} \Leftrightarrow S_i = 1) \wedge (\mathtt{VAR}_i > \mathtt{VAR}_{i+1} \Leftrightarrow S_i = 2)$.

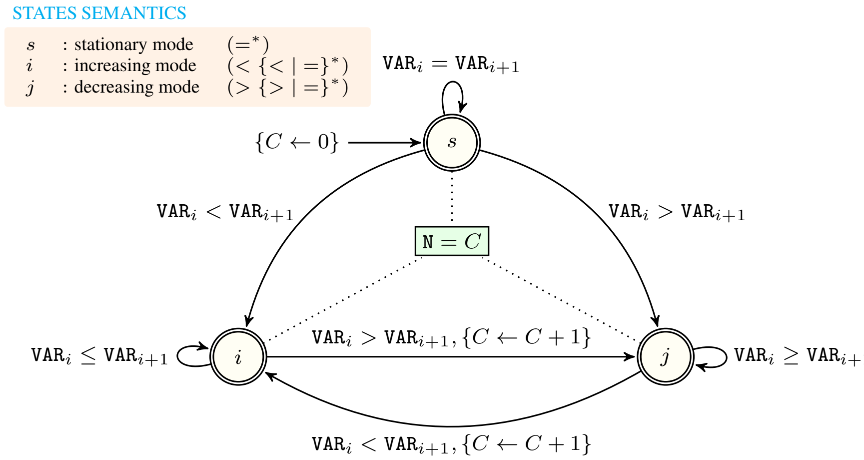


Figure 5.430: Automaton of the `inflexion` constraint (state s means that we are in *stationary* mode, state i means that we are in *increasing* mode, state j means that we are in *decreasing* mode, a new inflexion is detected each time we switch from increasing to decreasing mode – or conversely from decreasing to increasing mode – and the counter C is incremented accordingly)

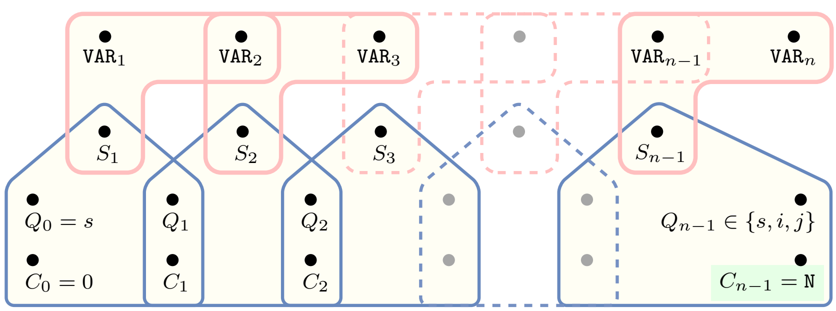


Figure 5.431: Hypergraph of the reformulation corresponding to the automaton of the `inflexion` constraint

Glue matrix where $\overrightarrow{C}$ and $\overleftarrow{C}$ resp. represent the counter value C at the end of a prefix and at the end of the corresponding reverse suffix that partitions the sequence VARIABLES.

	$s\,(=^*)$	$i\,(<\{<\mid=\}^*)$	$j\,(>\{>\mid=\}^*)$
$s\,(=^*)$	0	$\overleftarrow{C}$	$\overleftarrow{C}$
$i\,(<\{<\mid=\}^*)$	$\overrightarrow{C}$	$\overrightarrow{C}+1+\overleftarrow{C}$	$\overrightarrow{C}+\overleftarrow{C}$
$j\,(>\{>\mid=\}^*)$	$\overrightarrow{C}$	$\overrightarrow{C}+\overleftarrow{C}$	$\overrightarrow{C}+1+\overleftarrow{C}$

Figure 5.432: Glue matrix associated with the automaton of the `inflexion` constraint

`inflexion`$(\mathtt{N}=3, \langle 1,1,4,8,8,2,7,1\rangle)$

	1	1	4	8	8	2		2	7	1	
	=	<	<	=	>			>	<		
i	0	1	2	3	4	5		2	1	0	i
$\overrightarrow{Q_i}$	s	s	i	i	i	j		j	i	s	$\overleftarrow{Q_i}$
$\overrightarrow{C_i}$	0	0	0	0	0	1		1	0	0	$\overleftarrow{C_i}$
$\overrightarrow{\mathtt{N}_i}$	0	0	0	0	0	1		1	0	0	$\overleftarrow{\mathtt{N}_i}$

`inflexion`$\left(\begin{array}{c}\overrightarrow{\mathtt{N}_5}=1,\\ \langle 1,1,4,8,8,2\rangle\end{array}\right)$ `inflexion`$\left(\begin{array}{c}\overleftarrow{\mathtt{N}_2}=1,\\ \langle 1,7,2\rangle\end{array}\right)$

glue matrix entry associated with the state pair (j,j):

$$\mathtt{N}=\overrightarrow{\mathtt{C}_5}+1+\overleftarrow{\mathtt{C}_2}=1+1+1=3$$

Figure 5.433: Illustrating the use of the state pair (j,j) of the glue matrix for linking N with the counters variables obtained after reading the prefix $1,1,4,8,8,2$ and corresponding suffix $2,7,1$ of the sequence $1,1,4,8,8,2,7,1$; note that the suffix $2,7,1$ (in pink) is proceed in reverse order; the left (resp. right) table shows the initialisation (for $i=0$) and the evolution (for $i>0$) of the state of the automaton and its counter C upon reading the prefix $1,1,4,8,8,2$ (resp. the reverse suffix $1,7,2$).