5.209 k_same_modulo

Origin   Derived from same_modulo and from k_same.

Constraint   k_same_modulo(SETS, M)

Type   VARIABLES : collection(var−dvar)

Arguments   SETS : collection(set − VARIABLES)
                     M   : int

Restrictions   required(VARIABLES, var)
                        |VARIABLES| ≥ 1
                        required(SETS, set)
                        |SETS| > 1
                        same_size(SETS, set)
                        M > 0

Purpose   Given a collection of |SETS| sets, each containing the same number of domain variables, the k_same_modulo constraint forces a same_modulo constraint between each pair of consecutive sets.

Example
$$\left(\left\langle \begin{array}{l} \texttt{set} - \langle 1, 9, 1, 5, 2, 1 \rangle, \\ \texttt{set} - \langle 6, 4, 1, 1, 5, 5 \rangle, \\ \texttt{set} - \langle 1, 3, 4, 2, 8, 7 \rangle \end{array} \right\rangle, 3 \right)$$

The k_same_modulo constraint holds since:

- The first and second collections of variables are assigned 1 value in $\{0, 3, \ldots, 3 \cdot k\}$, 3 values in $\{1, 4, \ldots, 1 + 3 \cdot k\}$ and 2 values in $\{2, 5, \ldots, 2 + 3 \cdot k\}$.

- The second and third collections of variables are also assigned 1 value in $\{0, 3, \ldots, 3 \cdot k\}$, 3 values in $\{1, 4, \ldots, 1 + 3 \cdot k\}$ and 2 values in $\{2, 5, \ldots, 2 + 3 \cdot k\}$.

Typical |VARIABLES| > 1
                   M > 1

Symmetries
- Items of SETS are permutable.
- Items of SETS.set are permutable.
- An occurrence of a value u of SETS.set.var can be replaced by any other value v such that v is congruent to u modulo M.

Arg. properties
Contractible wrt. SETS.

See also   **common keyword:** k_same *(system of constraints)*.

                    implies: k_used_by_modulo.

                    part of system of constraints: same_modulo.

                    used in graph description: same_modulo.

Keywords   **characteristic of a constraint:** sort based reformulation, modulo.

                    combinatorial object: permutation.

                    constraint type: system of constraints, decomposition.

Arc input(s)	SETS
Arc generator	$PATH \mapsto \texttt{collection}(\texttt{set1}, \texttt{set2})$
Arc arity	2
Arc constraint(s)	$\texttt{same\_modulo}(\texttt{set1.set}, \texttt{set2.set}, \texttt{M})$
Graph property(ies)	$\textbf{NARC} = |\text{SETS}| - 1$

Graph model   Parts (A) and (B) of Figure 5.463 respectively show the initial and final graph associated with the **Example** slot. To each vertex corresponds a collection of variables, while to each arc corresponds a $\texttt{same\_modulo}$ constraint.
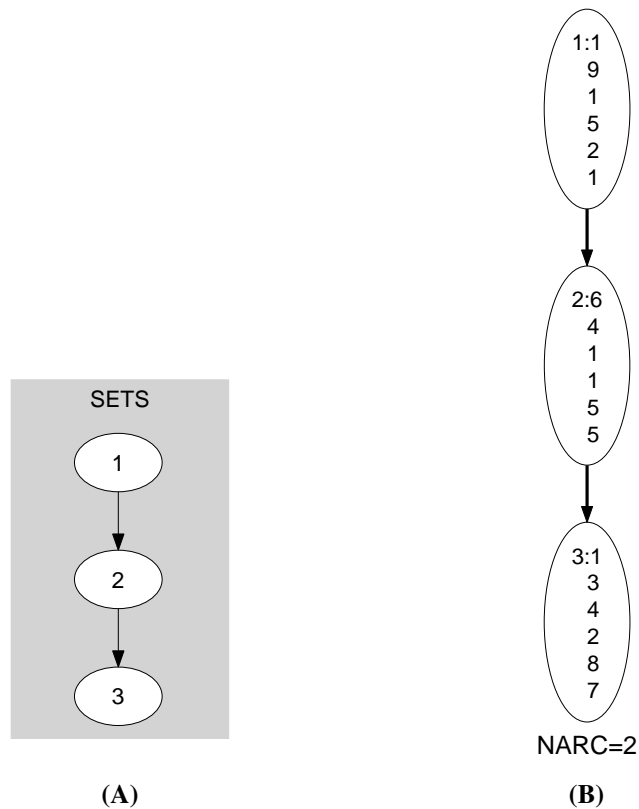

Figure 5.463: Initial and final graph of the $\texttt{k\_same\_modulo}$ constraint