5.214 k_used_by_partition

Origin   Derived from used_by_partition and from k_used_by.

Constraint   k_used_by_partition(SETS, PARTITIONS)

Types
VARIABLES : collection(var−dvar)
VALUES   : collection(val−int)

Arguments
SETS   : collection(set − VARIABLES)
PARTITIONS : collection(p − VALUES)

Restrictions
required(VARIABLES, var)
$|\text{VARIABLES}| \geq 1$
$|\text{VALUES}| \geq 1$
required(VALUES, val)
distinct(VALUES, val)
required(SETS, set)
$|\text{SETS}| > 1$
non_increasing_size(SETS, set)
required(PARTITIONS, p)
$|\text{PARTITIONS}| \geq 2$

Purpose   Given |SETS| sets of domain variables, the k_used_by_partition constraint forces a used_by_partition constraint between each pair of consecutive sets.

Example
$$\left(\begin{array}{l} \langle \text{set} - \langle 1, 9, 1, 6, 2, 3 \rangle, \text{set} - \langle 1, 3, 6, 6 \rangle, \text{set} - \langle 2, 2 \rangle \rangle, \\ \langle \text{p} - \langle 1, 3 \rangle, \text{p} - \langle 4 \rangle, \text{p} - \langle 2, 6 \rangle \rangle \end{array} \right)$$

The k_used_by_partition constraint holds since:

- The first collection of variables is assigned 3 values in $\{1, 3\}$, 0 value in $\{4\}$ and 2 values in $\{2, 6\}$, while the second collection of variables is assigned no more values in the previous three sets of values.

- The second collection of variables is assigned 2 values in $\{1, 3\}$, 0 value in $\{4\}$ and 2 values in $\{2, 6\}$, while the third collection of variables is assigned no more values in the previous three sets of values.

Typical $|\text{VARIABLES}| > 1$

Symmetries
- Items of `SETS` are permutable.
- Items of `SETS.set` are permutable.
- Items of `PARTITIONS` are permutable.
- Items of `PARTITIONS.p` are permutable.
- An occurrence of a value of `SETS.set.var` can be replaced by any other value that also belongs to the same partition of `PARTITIONS`.

Arg. properties

Contractible wrt. `SETS`.

See also
common keyword: `k_used_by` *(system of constraints)*.

implied by: `k_same_partition`.

part of system of constraints: `used_by_partition`.

used in graph description: `used_by_partition`.

Keywords
characteristic of a constraint: partition, sort based reformulation.

constraint type: system of constraints, decomposition.

Arc input(s)	SETS
Arc generator	$PATH \mapsto \text{collection}(\text{set1}, \text{set2})$
Arc arity	2
Arc constraint(s)	used_by_partition(set1.set, set2.set, PARTITIONS)
Graph property(ies)	**NARC**= |SETS| -1

Graph model   Parts (A) and (B) of Figure 5.468 respectively show the initial and final graph associated with the **Example** slot. To each vertex corresponds a collection of variables, while to each arc corresponds a used_by_partition constraint.
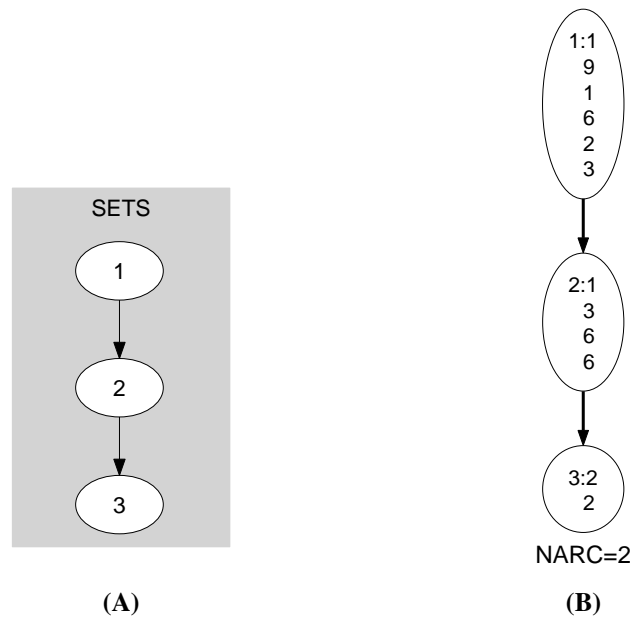

(A)   **(B)**

Figure 5.468: Initial and final graph of the k_used_by_partition constraint