5.216 length_last_sequence

DESCRIPTION LINKS AUTOMATON

Origin

Inspired by `stretch_path`

Constraint

`length_last_sequence(LEN, VARIABLES)`

Arguments

`LEN`	:	`dvar`
`VARIABLES`	:	`collection(var-dvar)`

Restrictions

`LEN` ≥ 0
`LEN` $\leq$ |`VARIABLES`|
`required(VARIABLES, var)`

Purpose

`LEN` is the length of the maximum sequence of variables that take the same value that contains the last variable of the collection `VARIABLES` (or 0 if the collection is empty).

Example

$(1, \langle 4, 4, 4, 5, 5, 4\rangle)$
$(6, \langle 4, 4, 4, 4, 4, 4\rangle)$
$(5, \langle 2, 4, 4, 4, 4, 4\rangle)$

The first `length_last_sequence` constraint holds since the sequence associated with the last value of the collection `VARIABLES` $= \langle 4, 4, 4, 5, 5, 4\rangle$ spans over a single variable.

Typical

`LEN` $<$ |`VARIABLES`|
|`VARIABLES`| > 1

Symmetry

All occurrences of two distinct values of `VARIABLES.var` can be swapped; all occurrences of a value of `VARIABLES.var` can be renamed to any unused value.

Arg. properties

Functional dependency: `LEN` determined by `VARIABLES`.

Reformulation

Without loss of generality let assume that the collection `VARIABLES` $= \langle V_1, V_2, \dots, V_n\rangle$ has more than one variable. By introducing $2 \cdot n - 1$ 0-1 variables, the `length_last_sequence(LEN, VARIABLES)` constraint can be expressed in term of $2 \cdot n - 1$ reified constraints and one arithmetic constraint (i.e., a `sum_ctr` constraint). We first introduce $n - 1$ variables that are respectively set to 1 if and only if two given consecutive variables of the collection `VARIABLES` are equal:

$B_{n-1,n} \Leftrightarrow V_{n-1} = V_n$,
$B_{n-2,n-1} \Leftrightarrow V_{n-2} = V_{n-1}$,
....
$B_{1,2} \Leftrightarrow V_1 = V_2$.

We then introduce n variables $A_n, A_{n-1}, \dots, A_1$ that are respectively associated to the different sliding sequences ending on the last variable of the sequence $V_1\ V_2\ \dots\ V_n$. Variable A_i is set to 1 if and only if $V_n = V_{n-1} = \cdots = V_i$:

$A_n = 1,$
$A_{n-1} \Leftrightarrow B_{n-1,n} \wedge A_n,$
$A_{n-2} \Leftrightarrow B_{n-2,n-1} \wedge A_{n-1},$
$\dots\dots\dots\dots\dots\dots\dots\dots\dots$
$A_1 \Leftrightarrow B_{1,2} \wedge A_2.$

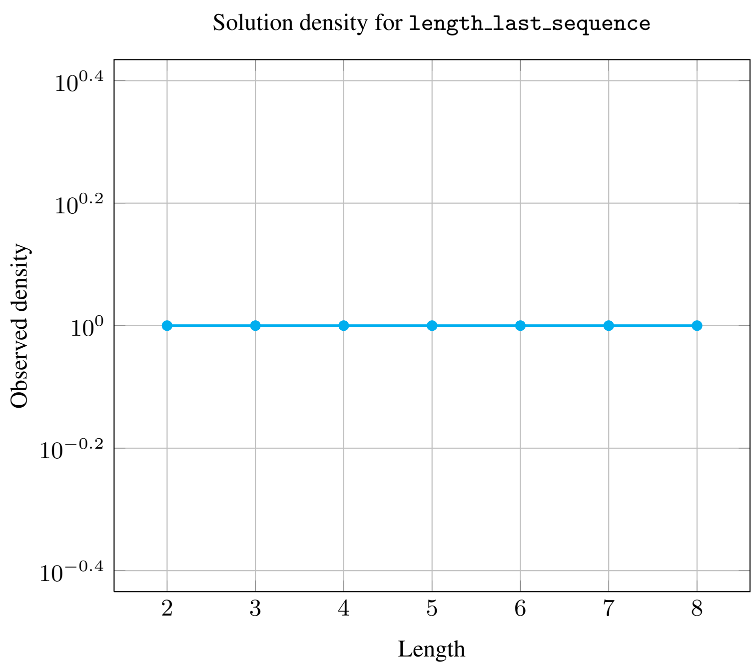
Finally we state the following arithmetic constraint:

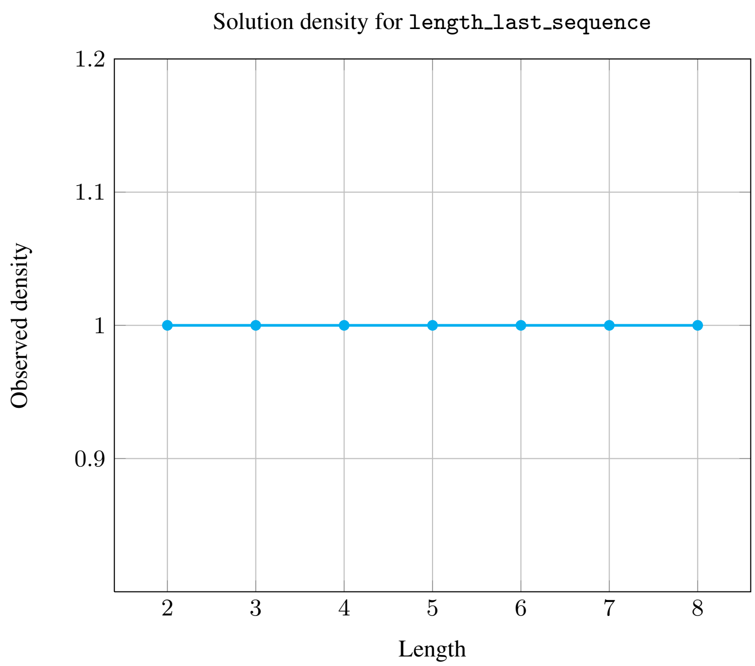
$\texttt{LEN} = A_n + A_{n-1} + \cdots + A_1.$

Counting

Length (n)	2	3	4	5	6	7	8
Solutions	9	64	625	7776	117649	2097152	43046721

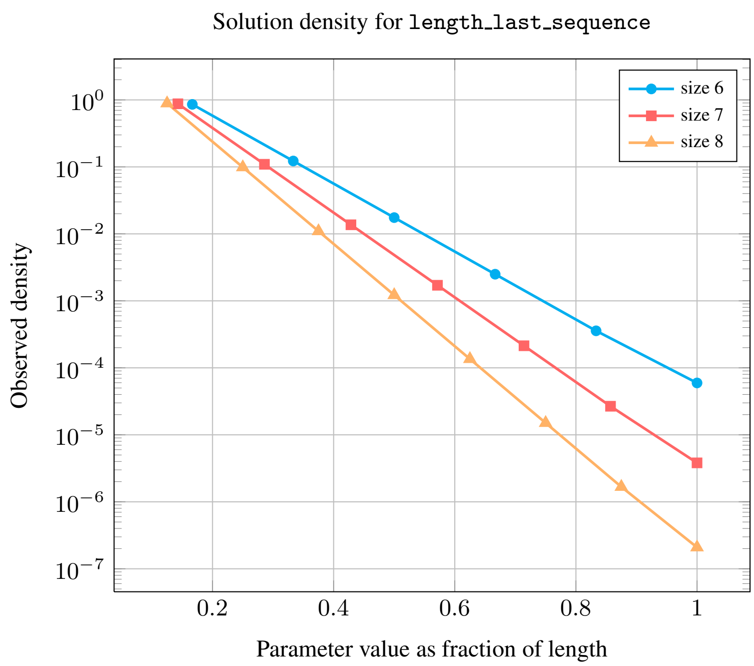
Number of solutions for `length_last_sequence`: domains $0..n$

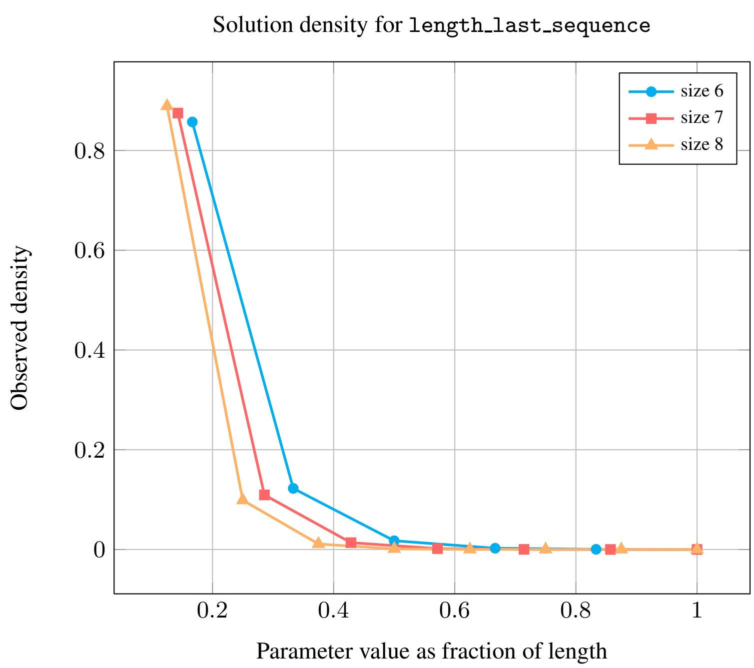




Length (n)		2	3	4	5	6	7	8
Total		9	64	625	7776	117649	2097152	43046721
Parameter value	1	6	48	500	6480	100842	1835008	38263752
	2	3	12	100	1080	14406	229376	4251528
	3	-	4	20	180	2058	28672	472392
	4	-	-	5	30	294	3584	52488
	5	-	-	-	6	42	448	5832
	6	-	-	-	-	7	56	648
	7	-	-	-	-	-	8	72
	8	-	-	-	-	-	-	9

Solution count for length_last_sequence: domains $0..n$





See also **common keyword:** `length_first_sequence` *(counting constraint,sequence)*.

Keywords **characteristic of a constraint:** automaton, automaton with counters.

combinatorial object: sequence.

constraint arguments: reverse of a constraint, pure functional dependency.

constraint network structure: sliding cyclic(1) constraint network(2).

constraint type: value constraint, counting constraint.

filtering: glue matrix.

modelling: functional dependency.

Automaton Figure 5.472 depicts the automaton associated with the `length_last_sequence` constraint. To each pair of consecutive variables $(\mathtt{VAR}_i, \mathtt{VAR}_{i+1})$ of the collection `VARIABLES` corresponds a signature variable S_i. The following signature constraint links $\mathtt{VAR}_i$, $\mathtt{VAR}_{i+1}$ and S_i: $\mathtt{VAR}_i = \mathtt{VAR}_{i+1} \Leftrightarrow S_i$.

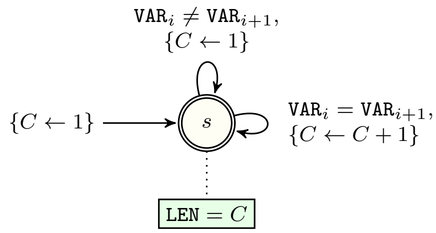


Figure 5.472: Automaton of the `length_last_sequence` constraint when $|\mathtt{VARIABLES}| \geq 2$

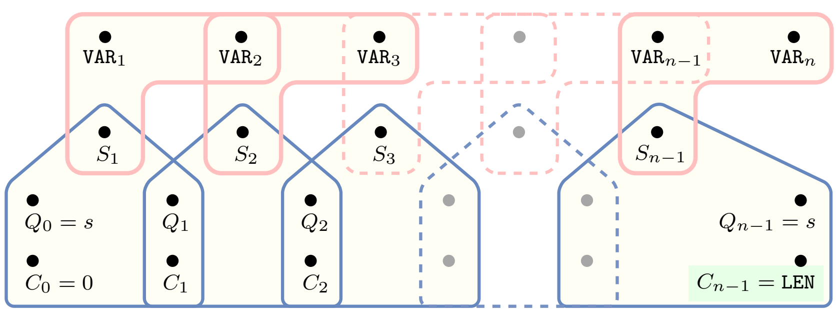


Figure 5.473: Hypergraph of the reformulation corresponding to the automaton of the `length_last_sequence` constraint

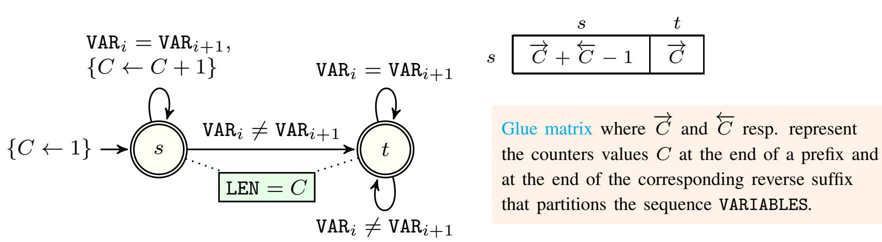


Figure 5.474: Automaton of the reverse of the `length_last_sequence` constraint (i.e., the `length_first_sequence` constraint) when $|\mathtt{VARIABLES}| \geq 2$ and corresponding glue matrix between `length_last_sequence` and its reverse `length_first_sequence`