5.232 lex_lesseq

DESCRIPTION LINKS GRAPH AUTOMATON

Origin CHIP

Constraint `lex_lesseq(VECTOR1, VECTOR2)`

Synonyms `lexeq`, `lex_chain`, `rel`, `lesseq`, `leq`, `lex_leq`.

Arguments

```
VECTOR1 : collection(var-dvar)
VECTOR2 : collection(var-dvar)
```

Restrictions

```
required(VECTOR1, var)
required(VECTOR2, var)
|VECTOR1| = |VECTOR2|
```

Purpose

VECTOR1 is *lexicographically less than or equal to* VECTOR2. Given two vectors, $\vec{X}$ and $\vec{Y}$ of n components, $\langle X_0, \ldots, X_{n-1}\rangle$ and $\langle Y_0, \ldots, Y_{n-1}\rangle$, $\vec{X}$ is *lexicographically less than or equal to* $\vec{Y}$ if and only if $n = 0$ or $X_0 < Y_0$ or $X_0 = Y_0$ and $\langle X_1, \ldots, X_{n-1}\rangle$ is *lexicographically less than or equal to* $\langle Y_1, \ldots, Y_{n-1}\rangle$.

Example

$$(\langle 5, 2, 3, 1\rangle, \langle 5, 2, 6, 2\rangle)$$
$$(\langle 5, 2, 3, 9\rangle, \langle 5, 2, 3, 9\rangle)$$

The `lex_lesseq` constraints associated with the first and second examples hold since:

- Within the first example VECTOR1 $= \langle 5, 2, 3, 1\rangle$ is lexicographically less than or equal to VECTOR2 $= \langle 5, 2, 6, 2\rangle$.
- Within the second example VECTOR1 $= \langle 5, 2, 3, 9\rangle$ is lexicographically less than or equal to VECTOR2 $= \langle 5, 2, 3, 9\rangle$.

Typical

$$|\mathtt{VECTOR1}| > 1$$
$$\vee \left(\begin{array}{l} |\mathtt{VECTOR1}| < 5, \\ \mathtt{nval}([\mathtt{VECTOR1.var}, \mathtt{VECTOR2.var}]) < 2 * |\mathtt{VECTOR1}| \end{array}\right)$$
$$\vee \left(\begin{array}{l} \mathtt{maxval}([\mathtt{VECTOR1.var}, \mathtt{VECTOR2.var}]) \leq 1, \\ 2 * |\mathtt{VECTOR1}| - \mathtt{max\_nvalue}([\mathtt{VECTOR1.var}, \mathtt{VECTOR2.var}]) > 2 \end{array}\right)$$

Symmetries

- `VECTOR1.var` can be decreased.
- `VECTOR2.var` can be increased.

Arg. properties

Suffix-contractible wrt. VECTOR1 and VECTOR2 (*remove items from same position*).

Remark

A *multiset ordering* constraint and its corresponding filtering algorithm are described in [174].

Algorithm

The first filtering algorithm maintaining arc-consistency for this constraint was presented in [173]. A second filtering algorithm maintaining arc-consistency and detecting entailment in a more eager way, was given in [96]. This second algorithm was derived from a deterministic finite automata. A third filtering algorithm extending the algorithm presented in [173] detecting entailment is given in the PhD thesis of Z. Kızıltan [239, page 95]. The previous thesis [239, pages 105–109] presents also a filtering algorithm handling the fact that a given variable has more than one occurrence. Finally, T. Frühwirth shows how to encode lexicographic ordering constraints within the context of CHR [175] in [176].

Reformulation

The following reformulations in term of arithmetic and/or logical expressions exist for enforcing the *lexicographically less than or equal to* constraint. The first one converts $\vec{X}$ and $\vec{Y}$ into numbers and post an inequality constraint. It assumes all components of $\vec{X}$ and $\vec{Y}$ to be within $[0, a-1]$:

$$a^{n-1}X_0 + a^{n-2}X_1 + \cdots + a^0 X_{n-1} \leq a^{n-1}Y_0 + a^{n-2}Y_1 + \cdots + a^0 Y_{n-1}$$

Since the previous reformulation can only be used with small values of n and a, W. Harvey came up with the following alternative model that maintains arc-consistency:

$$(X_0 < Y_0 + (X_1 < Y_1 + (\cdots + (X_{n-1} < Y_{n-1} + 1)\ldots))) = 1$$

Finally, the *lexicographically less than or equal to* constraint can be expressed as a conjunction or a disjunction of constraints:

$$\begin{array}{rl} X_0 \leq Y_0 & \wedge \\ (X_0 = Y_0) \Rightarrow X_1 \leq Y_1 & \wedge \\ (X_0 = Y_0 \wedge X_1 = Y_1) \Rightarrow X_2 \leq Y_2 & \wedge \\ \vdots & \\ (X_0 = Y_0 \wedge X_1 = Y_1 \wedge \cdots \wedge X_{n-2} = Y_{n-2}) \Rightarrow X_{n-1} \leq Y_{n-1} & \end{array}$$

$$\begin{array}{rl} X_0 < Y_0 & \vee \\ X_0 = Y_0 \wedge X_1 < Y_1 & \vee \\ X_0 = Y_0 \wedge X_1 = Y_1 \wedge X_2 < Y_2 & \vee \\ \vdots & \\ X_0 = Y_0 \wedge X_1 = Y_1 \wedge \cdots \wedge X_{n-2} = Y_{n-2} \wedge X_{n-1} \leq Y_{n-1} & \end{array}$$

When used separately, the two previous logical decompositions do not maintain arc-consistency.

Systems

lexEq in **Choco**, rel in **Gecode**, lex_lesseq in **MiniZinc**, lex_chain in **SICStus**.

Used in

`lex_between`, `lex_chain_greatereq`, `lex_chain_lesseq`, `ordered_atleast_nvector`, `ordered_atmost_nvector`, `ordered_nvector`.

See also

common keyword: `allperm`, `cond_lex_lesseq` *(lexicographic order)*, `lex2` *(matrix symmetry,lexicographic order)*, `lex_chain_greater`, `lex_chain_greatereq`, `lex_chain_less` *(lexicographic order)*, `lex_different` *(vector)*, `strict_lex2` *(matrix symmetry,lexicographic order)*.

implied by: `lex_equal`, `lex_less`, `lex_lesseq_allperm`.

implies (if swap arguments): `lex_greatereq`.

negation: `lex_greater`.

system of constraints: `lex_between`, `lex_chain_lesseq`.

Keywords

characteristic of a constraint: vector, automaton, automaton without counters, reified automaton constraint, derived collection.

constraint network structure: Berge-acyclic constraint network.

constraint type: order constraint.

filtering: duplicated variables, arc-consistency.

heuristics: heuristics and lexicographical ordering.

symmetry: symmetry, matrix symmetry, lexicographic order, multiset ordering.

Derived Collections

$$\text{col}\left(\begin{array}{l}\text{DESTINATION}-\text{collection}(\text{index}-\text{int},\text{x}-\text{int},\text{y}-\text{int}),\\ [\text{item}(\text{index}-0,\text{x}-0,\text{y}-0)]\end{array}\right)$$

$$\text{col}\left(\begin{array}{l}\text{COMPONENTS}-\text{collection}(\text{index}-\text{int},\text{x}-\text{dvar},\text{y}-\text{dvar}),\\ \left[\text{item}\left(\begin{array}{l}\text{index}-\text{VECTOR1.key},\\ \text{x}-\text{VECTOR1.var},\\ \text{y}-\text{VECTOR2.var}\end{array}\right)\right]\end{array}\right)$$

Arc input(s) COMPONENTS DESTINATION

Arc generator $PRODUCT(PATH, VOID) \mapsto \text{collection}(\text{item1}, \text{item2})$

Arc arity 2

Arc constraint(s)

$$\bigvee\left(\begin{array}{l}\text{item2.index} > 0 \wedge \text{item1.x} = \text{item1.y},\\ \bigwedge\left(\begin{array}{l}\text{item1.index} < |\text{VECTOR1}|,\\ \text{item2.index} = 0,\\ \text{item1.x} < \text{item1.y}\end{array}\right),\\ \bigwedge\left(\begin{array}{l}\text{item1.index} = |\text{VECTOR1}|,\\ \text{item2.index} = 0,\\ \text{item1.x} \leq \text{item1.y}\end{array}\right)\end{array}\right)$$

Graph property(ies) $\mathbf{PATH\_FROM\_TO}(\text{index}, 1, 0) = 1$

Graph model

Parts (A) and (B) of Figure 5.497 respectively show the initial and final graph associated with the first example of the **Example** slot. Since we use the **PATH_FROM_TO** graph property we show on the final graph the following information:

- The vertices, which respectively correspond to the start and the end of the required path, are stressed in bold.
- The arcs on the required path are also stressed in bold.

The vertices of the initial graph are generated in the following way:

- We create a vertex c_i for each pair of components that both have the same index i.
- We create an additional dummy vertex called d.

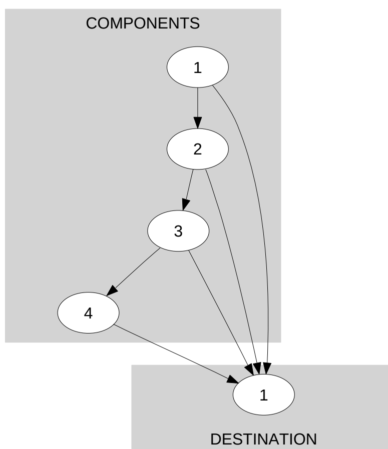
The arcs of the initial graph are generated in the following way:

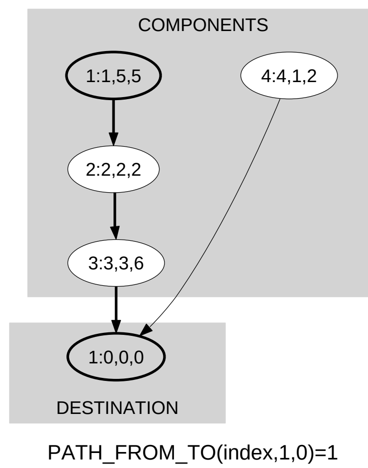
- We create an arc between c_i and d. When c_i was generated from the last components of both vectors We associate to this arc the arc constraint $\text{item}_1.x \leq \text{item}_2.y$; Otherwise we associate to this arc the arc constraint $\text{item}_1.x < \text{item}_2.y$;
- We create an arc between c_i and c_{i+1}. We associate to this arc the arc constraint $\text{item}_1.x = \text{item}_2.y$.

The `lex_lesseq` constraint holds when there exist a path from c_1 to d. This path can be interpreted as a maximum sequence of *equality* constraints on the prefix of both vectors, possibly followed by a *less than* constraint.

Signature

Since the maximum value returned by the graph property **PATH_FROM_TO** is equal to 1 we can rewrite $\mathbf{PATH\_FROM\_TO}(\text{index}, 1, 0) = 1$ to $\mathbf{PATH\_FROM\_TO}(\text{index}, 1, 0) \geq 1$. Therefore we simplify $\underline{\overline{\mathbf{PATH\_FROM\_TO}}}$ to $\overline{\mathbf{PATH\_FROM\_TO}}$.





(A) **(B)**

Figure 5.497: Initial and final graph of the `lex_lesseq` constraint

Automaton Figure 5.498 depicts the automaton associated with the `lex_lesseq` constraint. Let $\texttt{VAR1}_i$ and $\texttt{VAR2}_i$ respectively be the `var` attributes of the i^{th} items of the `VECTOR1` and the `VECTOR2` collections. To each pair $(\texttt{VAR1}_i, \texttt{VAR2}_i)$ corresponds a signature variable S_i as well as the following signature constraint: $(\texttt{VAR1}_i < \texttt{VAR2}_i \Leftrightarrow S_i = 1) \wedge (\texttt{VAR1}_i = \texttt{VAR2}_i \Leftrightarrow S_i = 2) \wedge (\texttt{VAR1}_i > \texttt{VAR2}_i \Leftrightarrow S_i = 3)$.

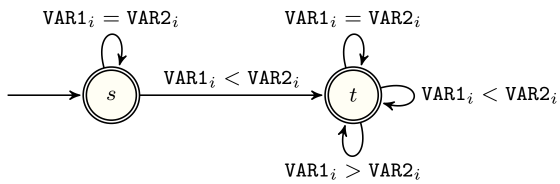


Figure 5.498: Automaton of the `lex_lesseq` constraint

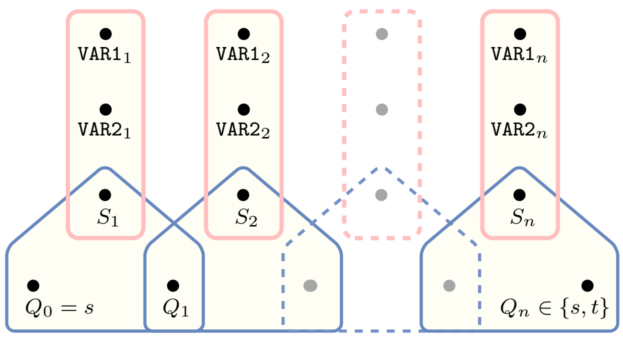


Figure 5.499: Hypergraph of the reformulation corresponding to the automaton of the `lex_lesseq` constraint