5.237 longest_increasing_sequence

DESCRIPTION LINKS AUTOMATON

Origin constraint on sequences

Constraint longest_increasing_sequence(L, VARIABLES)

Synonym size_longest_increasing_sequence.

Arguments

L	:	dvar
VARIABLES	:	collection(var−dvar)

Restrictions

$\texttt{L} \geq 0$
$\texttt{L} < \texttt{range}(\texttt{VARIABLES.var})$
required(VARIABLES, var)

Purpose L is the largest difference between the first and the last value of the maximum increasing sequences of the collection VARIABLES.
A sequence of consecutive variables $X_i, X_{i+1}, \ldots, X_j$ $(1 \leq i \leq j \leq |\texttt{VARIABLES}|)$ of the collection of variables VARIABLES is a *maximum increasing sequence* if all the following conditions simultaneously apply:

- $X_i \leq X_{i+1} \leq \cdots \leq X_j$,
- $i = 1$ or $X_{i-1} > X_i$,
- $i = |\texttt{VARIABLES}|$ or $X_j > X_{j+1}$.

Example $(7, \langle 10, 8, 8, 6, 4, 9, 11, 8\rangle)$
$(0, \langle 10, 8, 7, 5, 4, 3, 1, 0\rangle)$

Figure 5.507 gives a graphical representation of the first example of the **Example** slot with its two maximum increasing sequences in red of respective size 0 and 7. The corresponding longest_increasing_sequence constraint holds since its first argument L is fixed to the maximum size 7.

Typical

$\texttt{L} > 0$
$|\texttt{VARIABLES}| > 1$
$\texttt{nval}(\texttt{VARIABLES.var}) > 2$

Symmetry One and the same constant can be added to the var attribute of all items of VARIABLES.

Arg. properties

Functional dependency: L determined by VARIABLES.

Counting

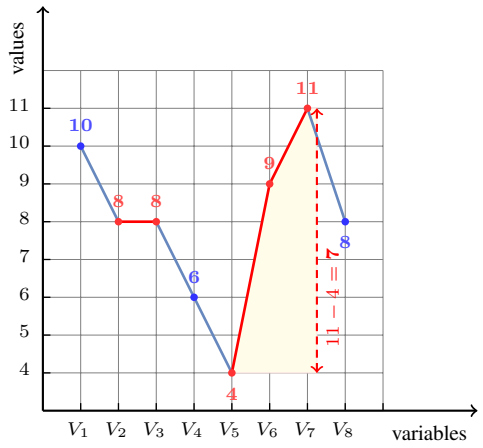
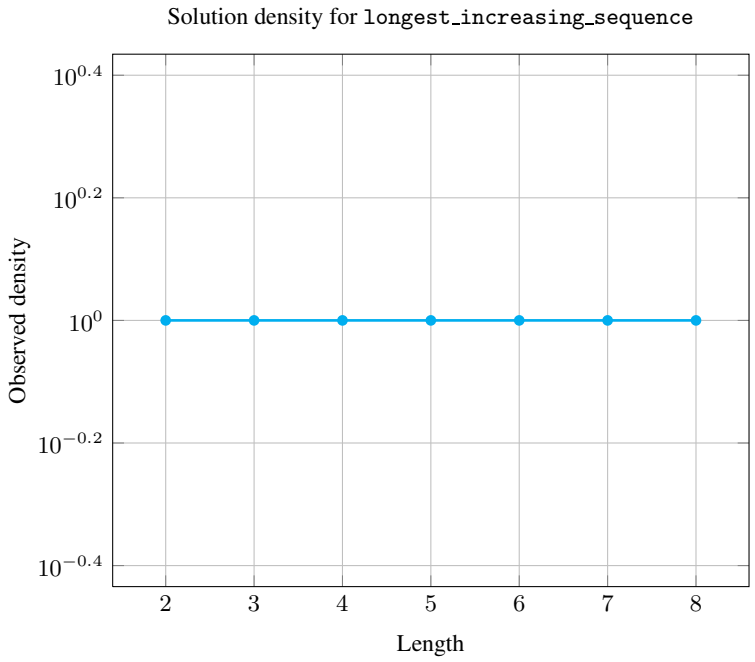
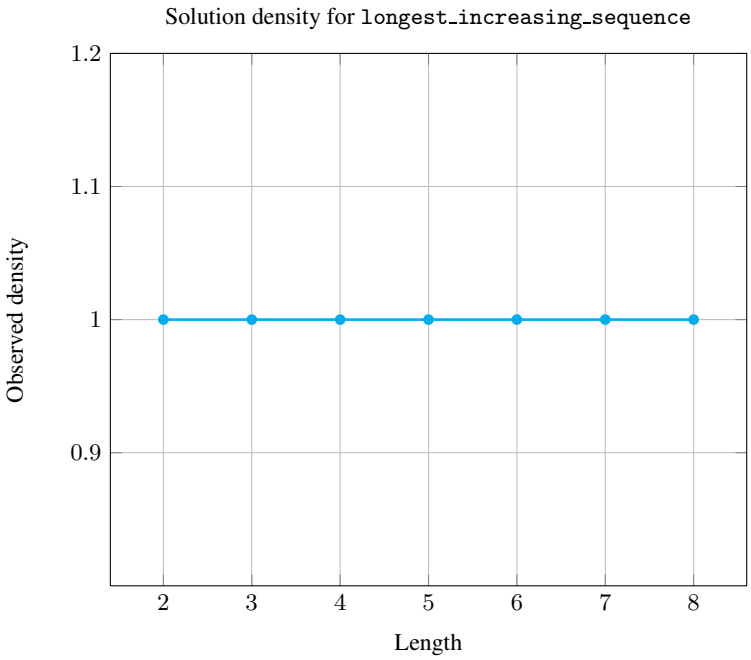


Figure 5.507: Illustration of the first example of the **Example** slot: a sequence of eight variables $V_1, V_2, V_3, V_4, V_5, V_6, V_7, V_8$ respectively fixed to values 10, 8, 8, 6, 4, 9, 11, 8 and its two maximum increasing sequences in red of respective size $8 - 8 = 0$ and $11 - 4 = 7$.

Length (n)	2	3	4	5	6	7	8
Solutions	9	64	625	7776	117649	2097152	43046721

Number of solutions for `longest_increasing_sequence`: domains $0..n$

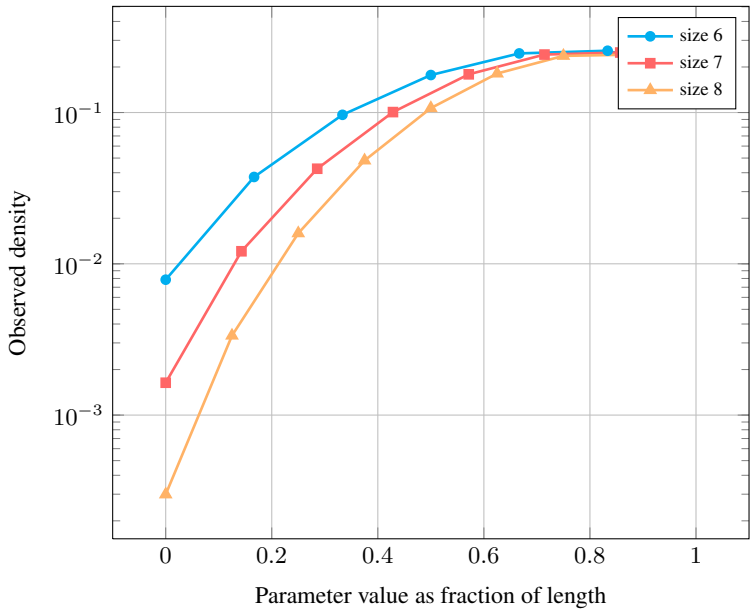




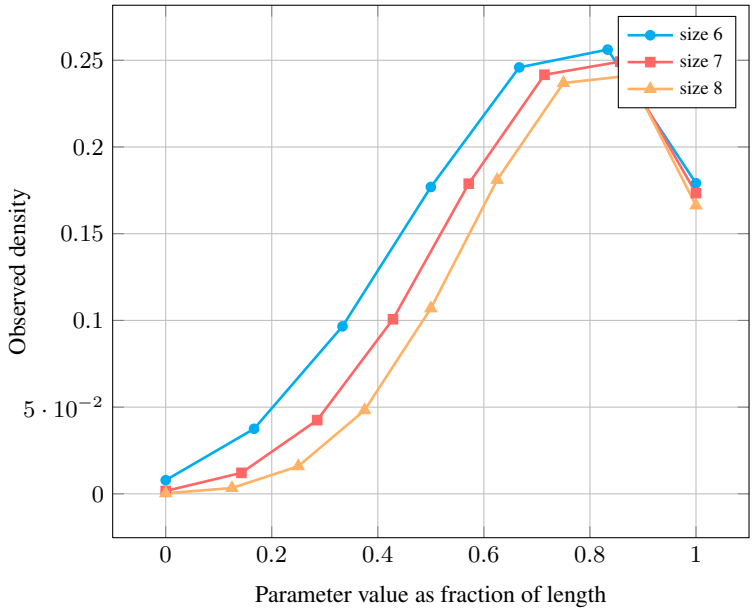
Length (n)		2	3	4	5	6	7	8
Total		9	64	625	7776	117649	2097152	43046721
Parameter value	0	6	20	70	252	924	3432	12870
	1	2	18	122	750	4412	25382	144314
	2	1	16	161	1398	11361	89132	685090
	3	-	10	162	1942	20816	211106	2074365
	4	-	-	110	2024	28930	375084	4603682
	5	-	-	-	1410	30134	506766	7792840
	6	-	-	-	-	21072	522648	10197174
	7	-	-	-	-	-	363602	10379696
	8	-	-	-	-	-	-	7156690

Solution count for `longest_increasing_sequence`: domains $0..n$

Solution density for `longest_increasing_sequence`



Solution density for `longest_increasing_sequence`



See also **common keyword:** `longest_decreasing_sequence`, `min_dist_between_inflexion` *(sequence)*.

Keywords

characteristic of a constraint: automaton, automaton with counters, automaton with same input symbol.

combinatorial object: sequence.

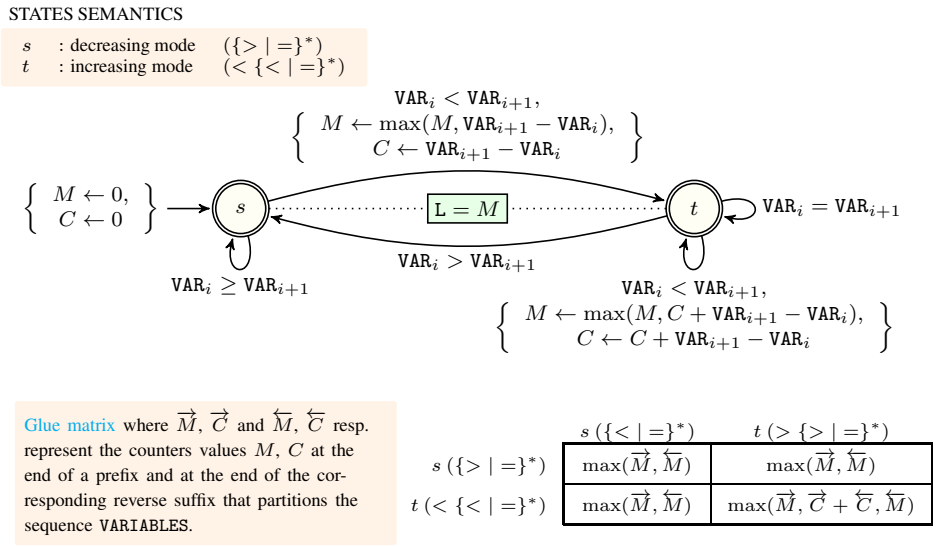
constraint arguments: reverse of a constraint, pure functional dependency.

filtering: glue matrix.

modelling: functional dependency.

Automaton

Figure 5.508 depicts the automaton associated with the `longest_increasing_sequence` constraint.



Glue matrix where $\overrightarrow{M}$, $\overrightarrow{C}$ and $\overleftarrow{M}$, $\overleftarrow{C}$ resp. represent the counters values M, C at the end of a prefix and at the end of the corresponding reverse suffix that partitions the sequence `VARIABLES`.

	s ($\{< \mid =\}^*$)	t ($> \{> \mid =\}^*$)
s ($\{> \mid =\}^*$)	$\max(\overrightarrow{M}, \overleftarrow{M})$	$\max(\overrightarrow{M}, \overleftarrow{M})$
t ($< \{< \mid =\}^*$)	$\max(\overrightarrow{M}, \overleftarrow{M})$	$\max(\overrightarrow{M}, \overrightarrow{C} + \overleftarrow{C}, \overleftarrow{M})$

Figure 5.508: Automaton of the `longest_increasing_sequence` constraint and its glue matrix (note that the reverse of the `longest_increasing_sequence` constraint is the `longest_decreasing_sequence` constraint)

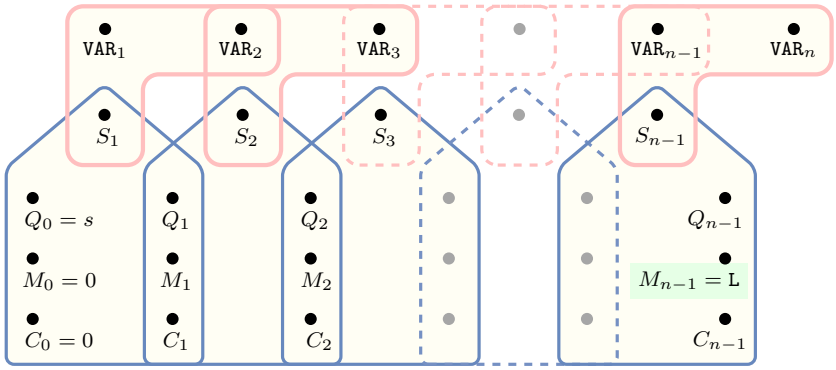


Figure 5.509: Hypergraph of the reformulation corresponding to the automaton of the `longest_increasing_sequence` constraint