5.240 max_decreasing_slope

DESCRIPTION LINKS AUTOMATON

Origin Motivated by time series.

Constraint max_decreasing_slope(MAX, VARIABLES)

Arguments

MAX : dvar
VARIABLES : collection(var−dvar)

Restrictions

$\texttt{MAX} \geq 0$
$\texttt{MAX} < \texttt{range}(\texttt{VARIABLES.var})$
required(VARIABLES, var)
$|\texttt{VARIABLES}| > 0$

Purpose

Given a sequence of variables VARIABLES $= V_1, V_2, \ldots, V_n$, sets MAX to 0 if $\nexists i \in [1, n-1] | V_i > V_{i+1}$, otherwise sets MAX to $\max_{i \in [1,n-1] | V_i > V_{i+1}} (V_i - V_{i+1})$.

Example

$(4, \langle 1, 1, 5, 8, 6, 2, 4, 1, 2 \rangle)$
$(0, \langle 1, 3, 5, 8 \rangle)$
$(8, \langle 3, 1, 9, 1 \rangle)$

The first max_decreasing_slope constraint holds since the sequence 1 1 5 8 6 2 4 1 2 contains two decreasing subsequences 8 6 2 and 4 1 and the maximum slope is equal to $\max(8-6, 6-2, 4-1) = 4$ as shown on Figure 5.511.

Typical

$\texttt{MAX} > 0$
$\texttt{MAX} < \texttt{range}(\texttt{VARIABLES.var}) - 1$
$|\texttt{VARIABLES}| > 2$
$\texttt{range}(\texttt{VARIABLES.var}) > 2$

Symmetry One and the same constant can be added to the var attribute of all items of VARIABLES.

Arg. properties Functional dependency: MAX determined by VARIABLES.

Usage Getting the maximum slope over the decreasing sequences of time series.

Counting

Length (n)	2	3	4	5	6	7	8
Solutions	9	64	625	7776	117649	2097152	43046721

Number of solutions for max_decreasing_slope: domains $0..n$

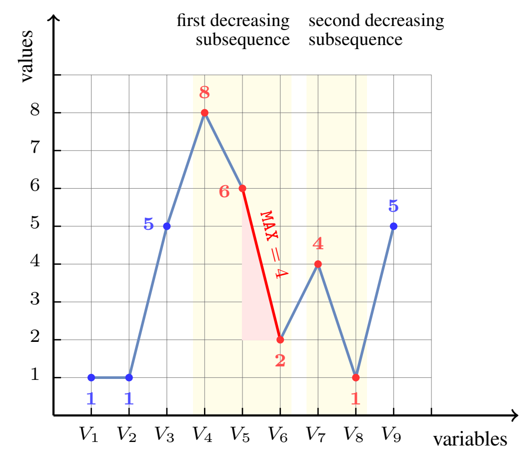
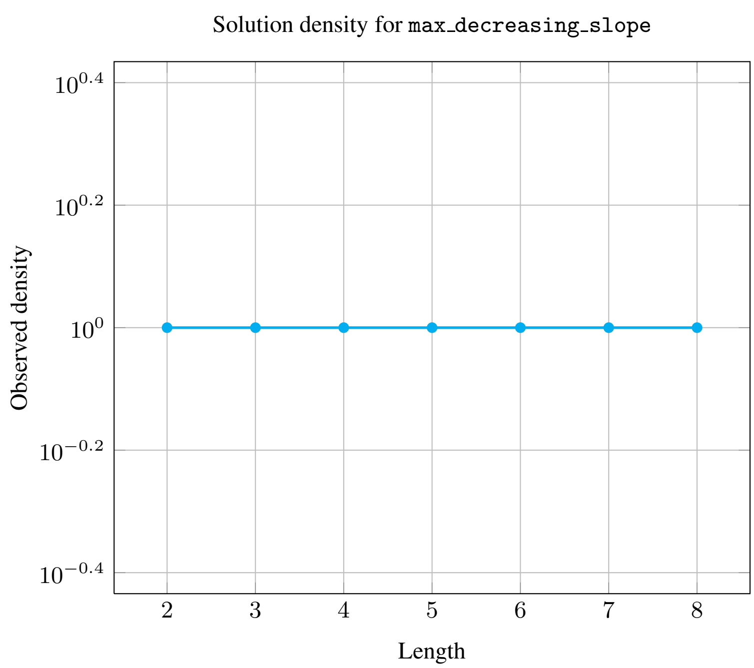
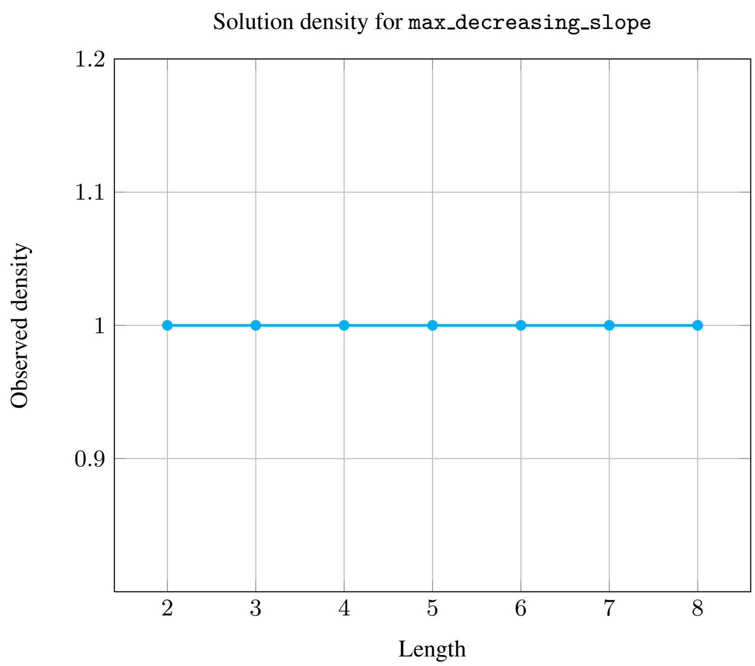


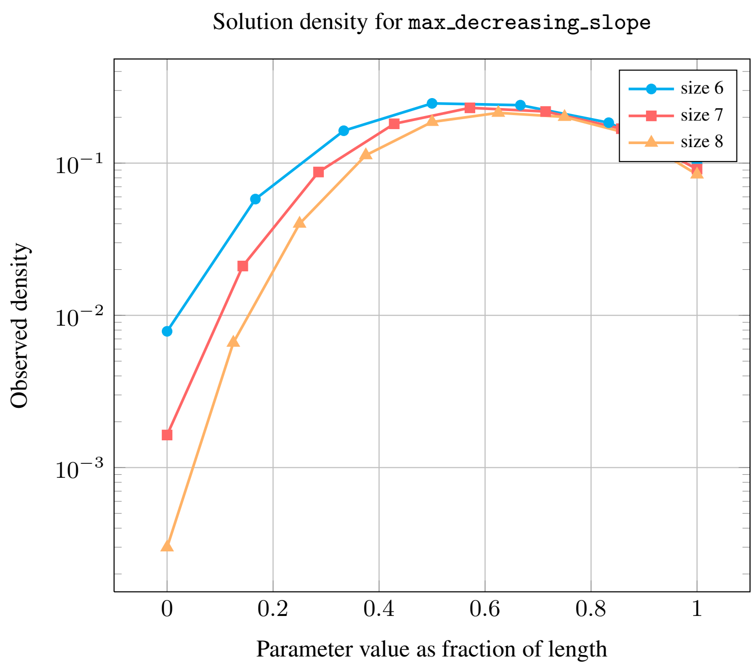
Figure 5.511: Illustration of the first example of the **Example** slot: a sequence of nine variables V_1, V_2, V_3, V_4, V_5, V_6, V_7, V_8, V_9 respectively fixed to values 1, 1, 5, 8, 6, 2, 4, 1, 5 and the corresponding maximum slope on the strictly decreasing subsequences 8 6 2 and 4 1 ($\mathtt{MAX} = 4$)

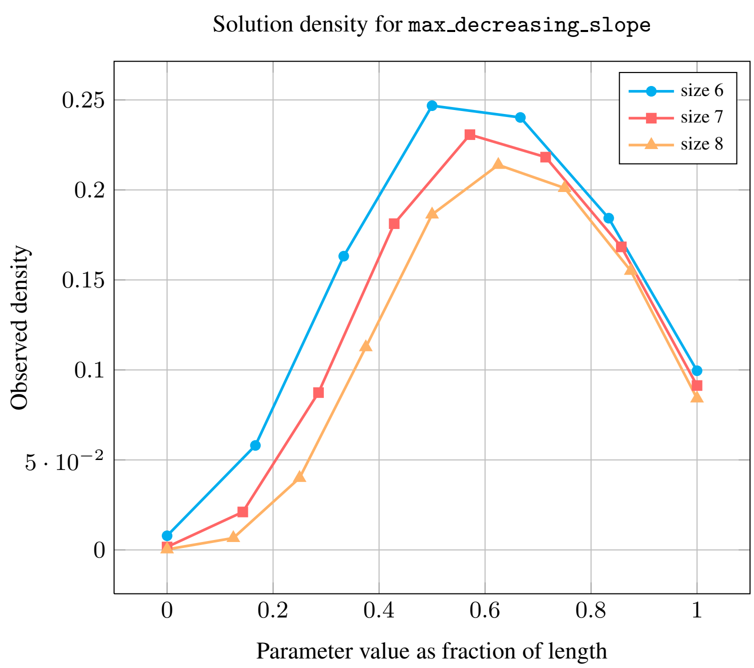




Length (n)		2	3	4	5	6	7	8
Total		9	64	625	7776	117649	2097152	43046721
Parameter value	0	6	20	70	252	924	3432	12870
	1	2	20	151	1036	6828	44220	284405
	2	1	16	188	1952	19200	183304	1721425
	3	-	8	142	2106	29035	380116	4847301
	4	-	-	74	1584	28266	483840	8021350
	5	-	-	-	846	21684	457632	9208124
	6	-	-	-	-	11712	353088	8654931
	7	-	-	-	-	-	191520	6673834
	8	-	-	-	-	-	-	3622481

Solution count for max_decreasing_slope: domains $0..n$





Keywords **characteristic of a constraint:** automaton, automaton with counters.

combinatorial object: sequence.

constraint arguments: reverse of a constraint, pure functional dependency.

filtering: glue matrix.

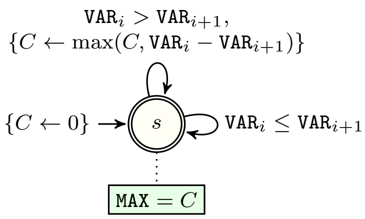
modelling: functional dependency.

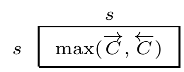
Cond. implications

- max_decreasing_slope(MAX, VARIABLES)
  with range(VARIABLES.var) = MAX + 1
  implies longest_decreasing_sequence(L, VARIABLES)
  when range(VARIABLES.var) = L + 1.

- max_decreasing_slope(MAX, VARIABLES)
  with MAX = 1
  implies min_decreasing_slope(MIN, VARIABLES)
  when MIN = 1.

Automaton Figure 5.512 depicts the automaton associated with the `max_decreasing_slope` constraint. To each pair of consecutive variables $(\mathtt{VAR}_i, \mathtt{VAR}_{i+1})$ of the collection `VARIABLES` corresponds a signature variable S_i. The following signature constraint links $\mathtt{VAR}_i$, $\mathtt{VAR}_{i+1}$ and S_i: $(\mathtt{VAR}_i \leq \mathtt{VAR}_{i+1} \Leftrightarrow S_i = 0) \wedge (\mathtt{VAR}_i > \mathtt{VAR}_{i+1} \Leftrightarrow S_i = 1)$.





Glue matrix where $\overrightarrow{C}$ and $\overleftarrow{C}$ resp. represent the counter value C at the end of a prefix and at the end of the corresponding reverse suffix that partitions the sequence `VARIABLES`.

Figure 5.512: Automaton for the `max_decreasing_slope` constraint and its glue matrix (note that the reverse of `max_decreasing_slope` is `max_increasing_slope`)