5.252 min_decreasing_slope

Origin   Motivated by time series.

Constraint   min_decreasing_slope(MIN, VARIABLES)

Arguments   MIN   : dvar
                       VARIABLES : collection(var−dvar)

Restrictions $\text{MIN} \geq 0$
                          $\text{MIN} < \text{range}(\text{VARIABLES.var})$
                          required(VARIABLES, var)
                          $|\text{VARIABLES}| > 0$

Purpose   Given a sequence of variables $\text{VARIABLES} = V_1, V_2, \ldots, V_n$, sets MIN to 0 if $\nexists i \in [1, n-1] | V_i > V_{i+1}$, otherwise sets MIN to $\min_{i \in [1,n-1] | V_i > V_{i+1}} (V_i - V_{i+1})$.

Example $(2, \langle 1, 1, 5, 8, 6, 2, 4, 1, 5 \rangle)$
                     $(0, \langle 1, 1, 1, 3, 4, 7, 7, 7, 9 \rangle)$
                     $(9, \langle 1, 1, 9, 0, 4, 7, 7, 7, 9 \rangle)$

The first min_decreasing_slope constraint holds since the sequence 1 1 5 8 6 2 4 1 5 contains two decreasing subsequences 8 6 2 and 4 1 and the minimum slope is equal to $\min(8 - 6, 6 - 2, 4 - 1) = 2$ as shown on Figure 5.529.

Typical $\text{MIN} > 1$
                     $|\text{VARIABLES}| > 2$
                     $\text{range}(\text{VARIABLES.var}) > 2$

Symmetry   One and the same constant can be added to the var attribute of all items of VARIABLES.

Arg. properties

Functional dependency: MIN determined by VARIABLES.

Usage   Getting the minimum slope over the decreasing sequences of time series.

Counting

Length (n)	2	3	4	5	6	7	8
Solutions	9	64	625	7776	117649	2097152	43046721

Number of solutions for min_decreasing_slope: domains $0..n$
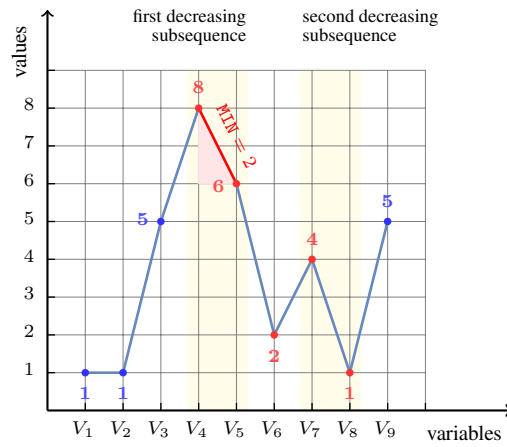
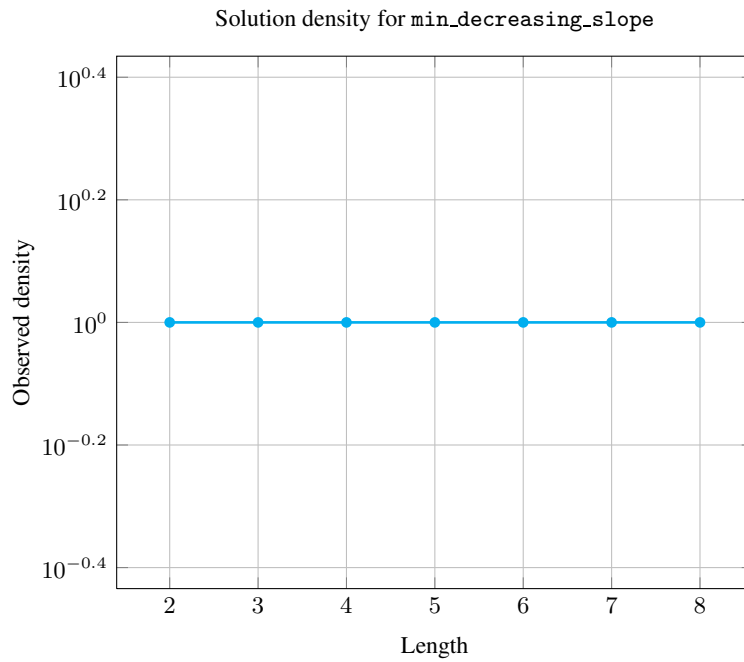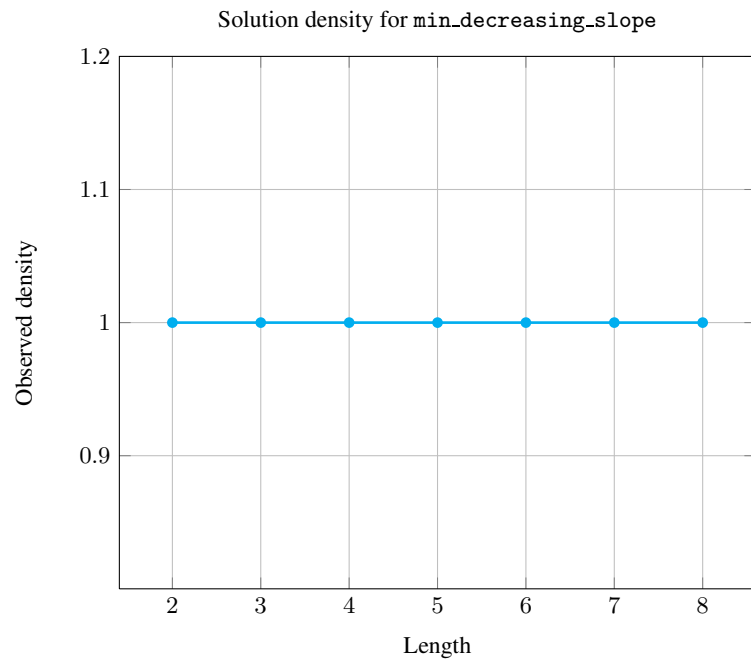Figure 5.529: Illustration of the first example of the **Example** slot: a sequence of nine variables $V_1, V_2, V_3, V_4, V_5, V_6, V_7, V_8, V_9$ respectively fixed to values $1, 1, 5, 8, 6, 2, 4, 1, 5$ and the corresponding minimum slope on the strictly decreasing subsequences $8\ 6\ 2$ and $4\ 1$ (MIN $= 2$)

Solution density for `min_decreasing_slope`

Solution density for `min_decreasing_slope`

Length (n)		2	3	4	5	6	7	8
Total		9	64	625	7776	117649	2097152	43046721
Parameter value	0	6	20	70	252	924	3432	12870
	1	2	22	256	3512	56537	1051936	22280084
	2	1	14	145	1864	28728	515372	10601773
	3	-	8	98	1062	14729	255076	5106480
	4	-	-	56	704	8853	133672	2475484
	5	-	-	-	382	5266	78198	1369232
	6	-	-	-	-	2612	41330	730161
	7	-	-	-	-	-	18136	341618
	8	-	-	-	-	-	-	129019

Solution count for `min_decreasing_slope`: domains $0..n$

Solution density for `min_decreasing_slope`

Solution density for `min_decreasing_slope`
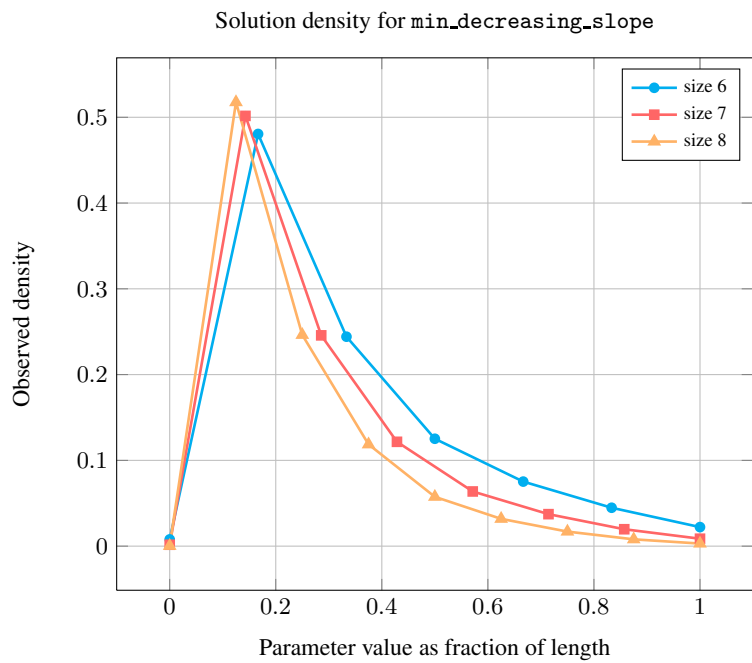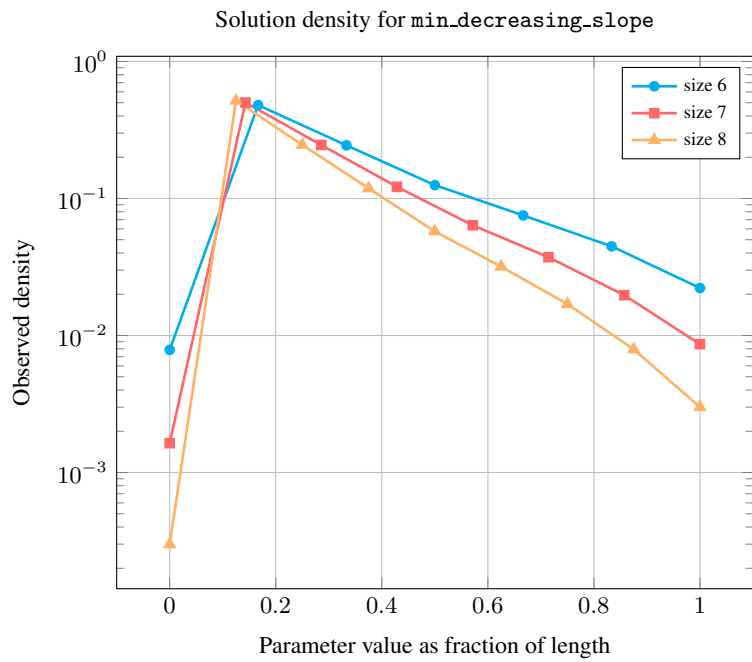
Keywords **characteristic of a constraint:** automaton, automaton with counters.

combinatorial object: sequence.

constraint arguments: reverse of a constraint, pure functional dependency.

filtering: glue matrix.

modelling: functional dependency.

Cond. implications   min_decreasing_slope(MIN, VARIABLES)
       with range(VARIABLES.var) = MIN + 1
     implies max_decreasing_slope(MAX, VARIABLES)
       when range(VARIABLES.var) = MAX + 1.

Automaton   Figure 5.530 depicts the automaton associated with the `min_decreasing_slope` constraint. To each pair of consecutive variables $(\text{VAR}_i, \text{VAR}_{i+1})$ of the collection `VARIABLES` corresponds a signature variable S_i. The following signature constraint links VAR_i, VAR_{i+1} and S_i: $(\text{VAR}_i \leq \text{VAR}_{i+1} \Leftrightarrow S_i = 0) \wedge (\text{VAR}_i > \text{VAR}_{i+1} \Leftrightarrow S_i = 1)$.

	s	t
s	0	$\overleftarrow{C}$
t	$\overrightarrow{C}$	$\min(\overrightarrow{C}, \overleftarrow{C})$
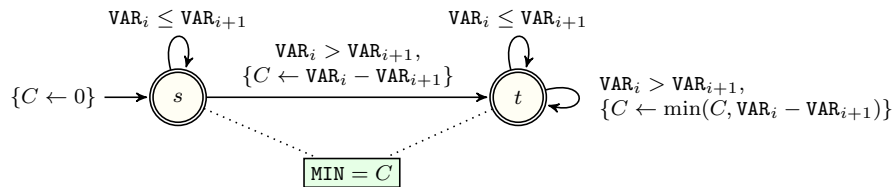
Glue matrix where $\overrightarrow{C}$ and $\overleftarrow{C}$ resp. represent the counter value C at the end of a prefix and at the end of the corresponding reverse suffix that partitions the sequence `VARIABLES`.

Figure 5.530: Automaton for the `min_decreasing_slope` constraint and its glue matrix (note that the reverse of `min_decreasing_slope` is `min_increasing_slope`)