5.254 min_increasing_slope

DESCRIPTION LINKS AUTOMATON

Origin

Motivated by time series.

Constraint

min_increasing_slope(MIN, VARIABLES)

Arguments

MIN : dvar
VARIABLES : collection(var−dvar)

Restrictions

MIN ≥ 0
MIN $<$range(VARIABLES.var)
required(VARIABLES, var)
$|$VARIABLES$| > 0$

Purpose

Given a sequence of variables VARIABLES $= V_1, V_2, \ldots, V_n$, sets MIN to 0 if $\nexists i \in [1, n-1] | V_i < V_{i+1}$, otherwise sets MIN to $\min_{i \in [1,n-1] | V_i < V_{i+1}} (V_{i+1} - V_i)$.

Example

$(3, \langle 1, 1, 5, 8, 6, 2, 2, 1, 5 \rangle)$
$(0, \langle 8, 8, 2, 0, 0 \rangle)$
$(9, \langle 1, 1, 0, 9, 6 \rangle)$

The first min_increasing_slope constraint holds since the sequence 1 1 5 8 6 2 2 1 5 contains two increasing subsequences 1 5 8 and 1 5 and the minimum slope is equal to $\min(5-1, 8-5, 5-1) = 3$ as shown on Figure 5.534.

Typical

MIN > 1
$|$VARIABLES$| > 2$
range(VARIABLES.var) > 2

Symmetry

One and the same constant can be added to the var attribute of all items of VARIABLES.

Arg. properties

Functional dependency: MIN determined by VARIABLES.

Usage

Getting the minimum slope over the increasing sequences of time series.

Counting

Length (n)	2	3	4	5	6	7	8
Solutions	9	64	625	7776	117649	2097152	43046721

Number of solutions for min_increasing_slope: domains $0..n$

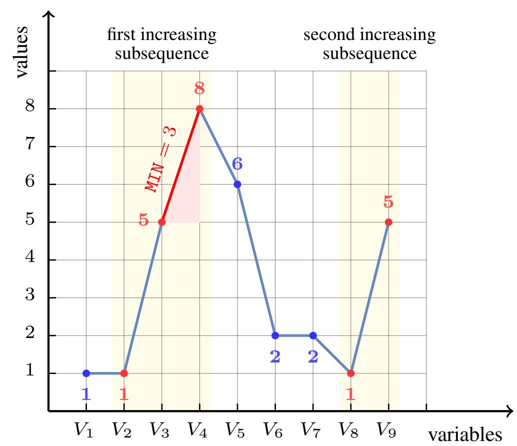
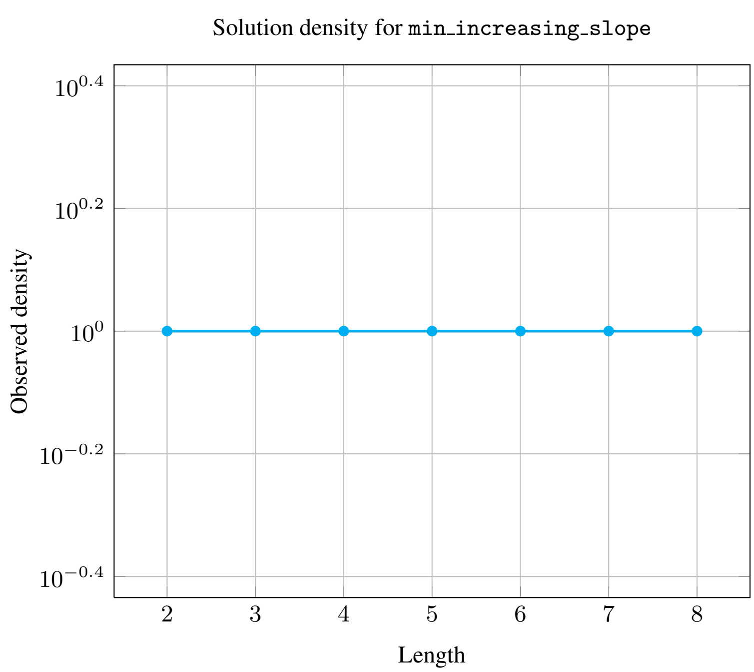
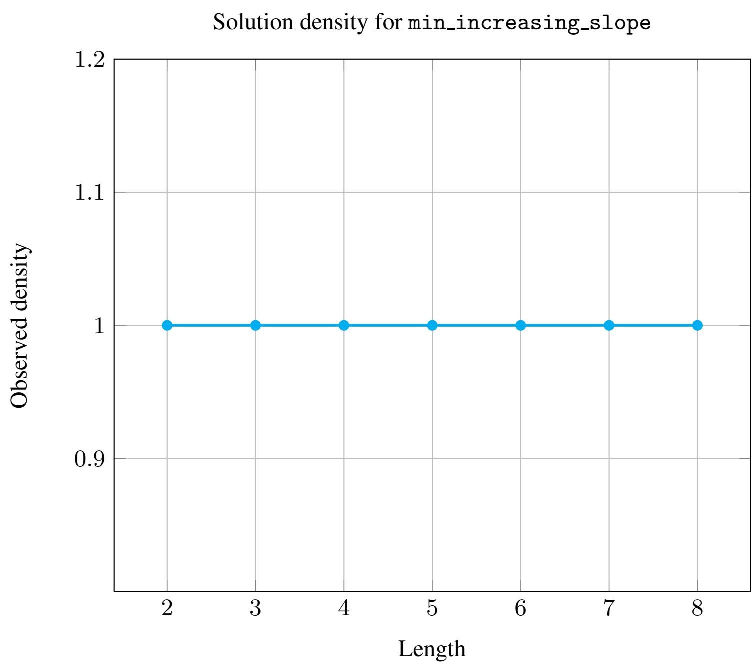


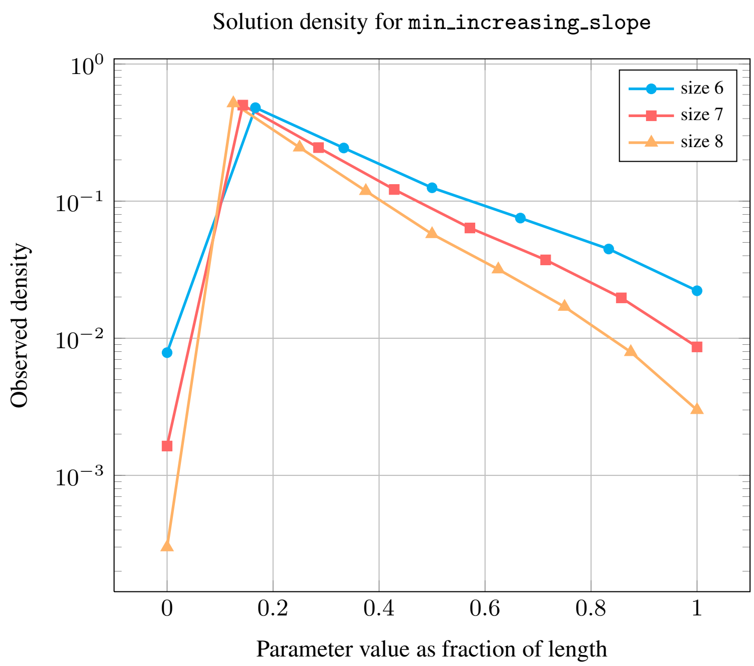
Figure 5.534: Illustration of the first example of the **Example** slot: a sequence of nine variables V_1, V_2, V_3, V_4, V_5, V_6, V_7, V_8, V_9 respectively fixed to values 1, 1, 5, 8, 6, 2, 2, 1, 5 and the corresponding minimum slope on the strictly increasing subsequences 1 5 8 and 1 5 (MIN $= 3$)

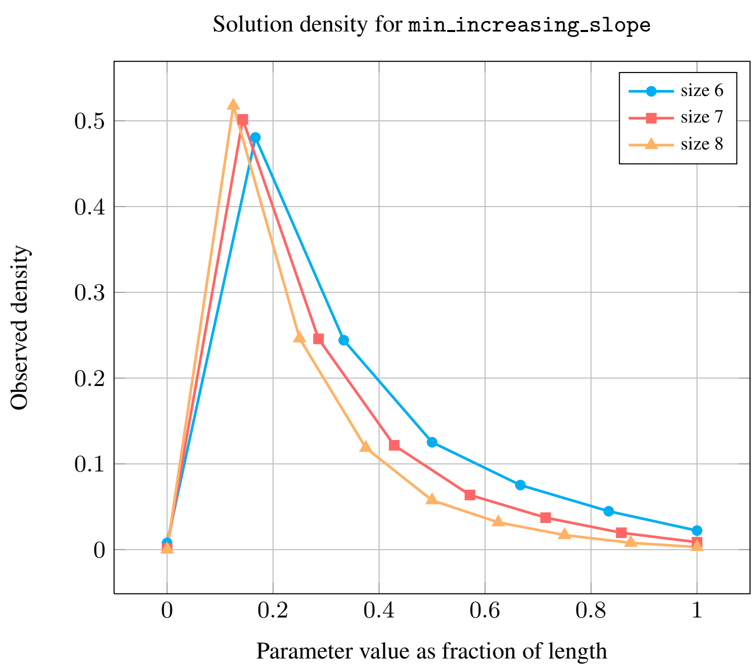




Length (n)		2	3	4	5	6	7	8
Total		9	64	625	7776	117649	2097152	43046721
Parameter value	0	6	20	70	252	924	3432	12870
	1	2	22	256	3512	56537	1051936	22280084
	2	1	14	145	1864	28728	515372	10601773
	3	-	8	98	1062	14729	255076	5106480
	4	-	-	56	704	8853	133672	2475484
	5	-	-	-	382	5266	78198	1369232
	6	-	-	-	-	2612	41330	730161
	7	-	-	-	-	-	18136	341618
	8	-	-	-	-	-	-	129019

Solution count for min_increasing_slope: domains $0..n$





Keywords **characteristic of a constraint:** automaton, automaton with counters.

combinatorial object: sequence.

constraint arguments: reverse of a constraint, pure functional dependency.

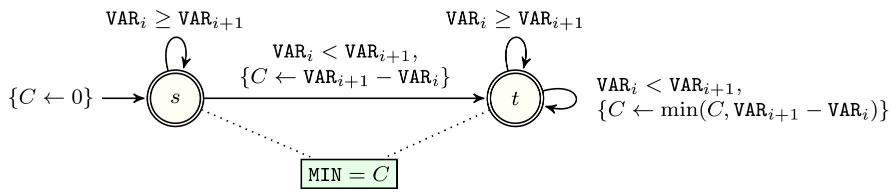
filtering: glue matrix.

modelling: functional dependency.

Cond. implications

`min_increasing_slope`(`MIN`, `VARIABLES`)
with `range`(`VARIABLES.var`) = `MIN` + 1
implies `max_increasing_slope`(`MAX`, `VARIABLES`)
when `range`(`VARIABLES.var`) = `MAX` + 1.

Automaton Figure 5.535 depicts the automaton associated with the `min_increasing_slope` constraint. To each pair of consecutive variables $(\texttt{VAR}_i, \texttt{VAR}_{i+1})$ of the collection `VARIABLES` corresponds a signature variable S_i. The following signature constraint links $\texttt{VAR}_i$, $\texttt{VAR}_{i+1}$ and S_i: $(\texttt{VAR}_i \geq \texttt{VAR}_{i+1} \Leftrightarrow S_i = 0) \wedge (\texttt{VAR}_i < \texttt{VAR}_{i+1} \Leftrightarrow S_i = 1)$.



Glue matrix where $\overrightarrow{C}$ and $\overleftarrow{C}$ resp. represent the counter value C at the end of a prefix and at the end of the corresponding reverse suffix that partitions the sequence `VARIABLES`.

	s	t
s	0	$\overleftarrow{C}$
t	$\overrightarrow{C}$	$\min(\overrightarrow{C}, \overleftarrow{C})$

Figure 5.535: Automaton for the `min_increasing_slope` constraint and its glue matrix (note that the reverse of `min_increasing_slope` is `min_decreasing_slope`)