5.266 minimum_weight_alldifferent

DESCRIPTION LINKS GRAPH

Origin [171]

Constraint minimum_weight_alldifferent(VARIABLES, MATRIX, COST)

Synonyms minimum_weight_alldiff, minimum_weight_alldistinct, min_weight_alldiff, min_weight_alldifferent, min_weight_alldistinct.

Arguments

VARIABLES	:	collection(var−dvar)
MATRIX	:	collection(i−int, j−int, c−int)
COST	:	dvar

Restrictions

$|\mathtt{VARIABLES}| > 0$
required(VARIABLES, var)
$\mathtt{VARIABLES.var} \geq 1$
$\mathtt{VARIABLES.var} \leq |\mathtt{VARIABLES}|$
required(MATRIX, [i, j, c])
increasing_seq(MATRIX, [i, j])
$\mathtt{MATRIX.i} \geq 1$
$\mathtt{MATRIX.i} \leq |\mathtt{VARIABLES}|$
$\mathtt{MATRIX.j} \geq 1$
$\mathtt{MATRIX.j} \leq |\mathtt{VARIABLES}|$
$|\mathtt{MATRIX}| = |\mathtt{VARIABLES}| * |\mathtt{VARIABLES}|$

Purpose

All variables of the VARIABLES collection should take a distinct value located within interval $[1, |\mathtt{VARIABLES}|]$. In addition COST is equal to the sum of the costs associated with the fact that we assign value i to variable j. These costs are given by the matrix MATRIX.

Example

$$\left(\begin{array}{l} \langle 2, 3, 1, 4\rangle, \\ \left\langle\begin{array}{lll} \mathtt{i}-1 & \mathtt{j}-1 & \mathtt{c}-4, \\ \mathtt{i}-1 & \mathtt{j}-2 & \mathtt{c}-1, \\ \mathtt{i}-1 & \mathtt{j}-3 & \mathtt{c}-7, \\ \mathtt{i}-1 & \mathtt{j}-4 & \mathtt{c}-0, \\ \mathtt{i}-2 & \mathtt{j}-1 & \mathtt{c}-1, \\ \mathtt{i}-2 & \mathtt{j}-2 & \mathtt{c}-0, \\ \mathtt{i}-2 & \mathtt{j}-3 & \mathtt{c}-8, \\ \mathtt{i}-2 & \mathtt{j}-4 & \mathtt{c}-2, \\ \mathtt{i}-3 & \mathtt{j}-1 & \mathtt{c}-3, \\ \mathtt{i}-3 & \mathtt{j}-2 & \mathtt{c}-2, \\ \mathtt{i}-3 & \mathtt{j}-3 & \mathtt{c}-1, \\ \mathtt{i}-3 & \mathtt{j}-4 & \mathtt{c}-6, \\ \mathtt{i}-4 & \mathtt{j}-1 & \mathtt{c}-0, \\ \mathtt{i}-4 & \mathtt{j}-2 & \mathtt{c}-0, \\ \mathtt{i}-4 & \mathtt{j}-3 & \mathtt{c}-6, \\ \mathtt{i}-4 & \mathtt{j}-4 & \mathtt{c}-5 \end{array}\right\rangle, 17 \end{array}\right)$$

The `minimum_weight_alldifferent` constraint holds since the cost 17 corresponds to the sum MATRIX$[(1-1)\cdot 4+2]$.c+MATRIX$[(2-1)\cdot 4+3]$.c+MATRIX$[(3-1)\cdot 4+1]$.c+MATRIX$[(4-1)\cdot 4+4]$.c = MATRIX[2].c+MATRIX[7].c+MATRIX[9].c+MATRIX[16].c = $1+8+3+5$.

All solutions Figure 5.569 gives all solutions to the following non ground instance of the `minimum_weight_alldifferent` constraint:
$V_1 \in [2,4]$, $V_2 \in [2,3]$, $V_3 \in [1,6]$, $V_4 \in [2,5]$, $V_5 \in [2,3]$, $V_6 \in [1,6]$, $C \in [0,25]$,
`minimum_weight_alldifferent`$(\langle V_1, V_2, V_3, V_4, V_5, V_6\rangle,$
$\langle 1\,1\,5,\ 1\,2\,0,\ 1\,3\,1,\ 1\,4\,1,\ 1\,5\,3,\ 1\,6\,0,$
$2\,1\,2,\ 2\,2\,7,\ 2\,3\,0,\ 2\,4\,2,\ 2\,5\,5,\ 2\,6\,1,$
$3\,1\,3,\ 3\,2\,3,\ 3\,3\,6,\ 3\,4\,6,\ 3\,5\,0,\ 3\,6\,9,$
$4\,1\,4,\ 4\,2\,3,\ 4\,3\,0,\ 4\,4\,0,\ 4\,5\,0,\ 4\,6\,2,$
$5\,1\,2,\ 5\,2\,0,\ 5\,3\,6,\ 5\,4\,3,\ 5\,5\,7,\ 5\,6\,2,$
$6\,1\,5,\ 6\,2\,4,\ 6\,3\,5,\ 6\,4\,4,\ 6\,5\,5,\ 6\,6\,4\rangle, C)$.

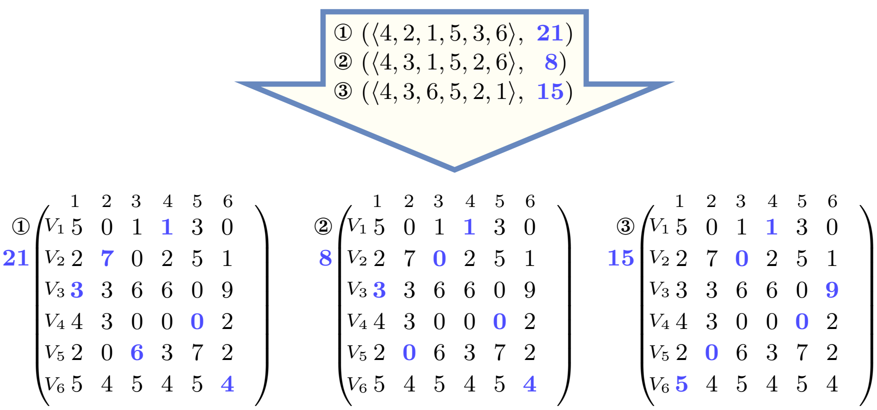


Figure 5.569: All solutions corresponding to the non ground example of the `minimum_weight_alldifferent` constraint of the **All solutions** slot

Typical

$|$VARIABLES$| > 1$
range(MATRIX.c) > 1
MATRIX.c > 0

Arg. properties

Functional dependency: COST determined by VARIABLES and MATRIX.

Algorithm The Hungarian method for the assignment problem [243] can be used for evaluating the bounds of the COST variable. A filtering algorithm is described in [377]. It can be used for handling both side of the `minimum_weight_alldifferent` constraint:

- Evaluating a lower bound of the COST variable and pruning the variables of the VARIABLES collection in order to not exceed the maximum value of COST.
- Evaluating an upper bound of the COST variable and pruning the variables of the VARIABLES collection in order to not be under the minimum value of COST.

Systems all_different in **SICStus**, all_distinct in **SICStus**.

See also **attached to cost variant:** `alldifferent`.

common keyword: `global_cardinality_with_costs` *(cost filtering constraint,weighted assignment)*, `sum_of_weights_of_distinct_values` *(weighted assignment)*, `weighted_partial_alldiff` *(cost filtering constraint,weighted assignment)*.

Keywords **application area:** assignment.

characteristic of a constraint: core.

filtering: cost filtering constraint, Hungarian method for the assignment problem.

final graph structure: one_succ.

modelling: cost matrix, functional dependency.

problems: weighted assignment.

Arc input(s) VARIABLES

Arc generator $CLIQUE \mapsto$ collection(variables1, variables2)

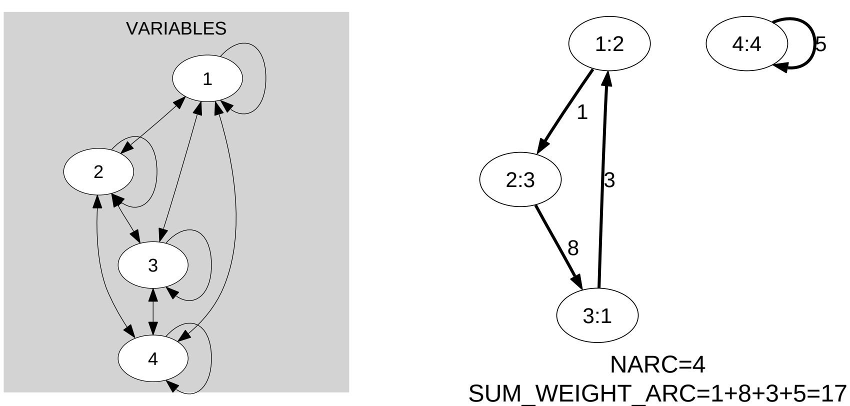
Arc arity 2

Arc constraint(s) variables1.var = variables2.key

Graph property(ies)
- **NTREE** = 0
- **SUM_WEIGHT_ARC**$\left(\texttt{MATRIX}\left[\sum\left(\begin{array}{l}(\texttt{variables1.key}-1)*|\texttt{VARIABLES}|,\\ \texttt{variables1.var}\end{array}\right)\right].\texttt{c}\right) = \texttt{COST}$

Graph model Since each variable takes one value, and because of the arc constraint variables1 = variables.key, each vertex of the initial graph belongs to the final graph and has exactly one successor. Therefore the sum of the out-degrees of the vertices of the final graph is equal to the number of vertices of the final graph. Since the sum of the in-degrees is equal to the sum of the out-degrees, it is also equal to the number of vertices of the final graph. Since **NTREE** = 0, each vertex of the final graph belongs to a circuit. Therefore each vertex of the final graph has at least one predecessor. Since we saw that the sum of the in-degrees is equal to the number of vertices of the final graph, each vertex of the final graph has exactly one predecessor. We conclude that the final graph consists of a set of vertex-disjoint elementary circuits.

Finally the graph constraint expresses that the COST variable is equal to the sum of the elementary costs associated with each variable-value assignment. All these elementary costs are recorded in the MATRIX collection. More precisely, the cost c_{ij} is recorded in the attribute c of the $((i-1)\cdot|\texttt{VARIABLES})|+j)^{th}$ entry of the MATRIX collection. This is ensured by the increasing restriction that enforces that the items of the MATRIX collection are sorted in lexicographically increasing order according to attributes i and j.



(A) **(B)**

Figure 5.570: Initial and final graph of the minimum_weight_alldifferent constraint

Parts (A) and (B) of Figure 5.570 respectively show the initial and final graph associated with the **Example** slot. Since we use the **SUM_WEIGHT_ARC** graph property, the arcs of the final graph are stressed in bold. We also indicate their corresponding weight.