

5.275 next_element

	DESCRIPTION	LINKS	GRAPH	AUTOMATON
Origin	N. Beldiceanu			
Constraint	<code>next_element(THRESHOLD, INDEX, TABLE, VAL)</code>			
Arguments	THRESHOLD : <code>dvar</code> INDEX : <code>dvar</code> TABLE : <code>collection(index-int, value-dvar)</code> VAL : <code>dvar</code>			
Restrictions	$\text{INDEX} \geq 1$ $\text{INDEX} \leq \text{TABLE} $ $\text{THRESHOLD} < \text{INDEX}$ <code>required(TABLE, [index, value])</code> $ \text{TABLE} > 0$ $\text{TABLE.index} \geq 1$ $\text{TABLE.index} \leq \text{TABLE} $ <code>distinct(TABLE, index)</code>			
Purpose	<div style="border: 1px solid pink; padding: 5px;"> INDEX is the smallest entry of TABLE strictly greater than THRESHOLD containing value VAL. </div>			
Example	<div style="border: 1px solid blue; padding: 10px; display: inline-block;"> $\left(2, 3, \left\langle \begin{array}{ll} \text{index} - 1 & \text{value} - 1, \\ \text{index} - 2 & \text{value} - 8, \\ \text{index} - 3 & \text{value} - 9, \\ \text{index} - 4 & \text{value} - 5, \\ \text{index} - 5 & \text{value} - 9 \end{array} \right\rangle, 9 \right)$ </div> <p>The <code>next_element</code> constraint holds since 3 is the smallest entry located after entry 2 that contains value 9.</p>			
Typical	$ \text{TABLE} > 1$ <code>range(TABLE.value) > 1</code>			
Usage	Originally introduced for modelling the fact that a nucleotide has to be consumed as soon as possible at cycle INDEX after a given cycle represented by variable THRESHOLD.			
See also	related: <code>minimum_greater_than</code> (<i>identify an element in a table</i>), <code>next_greater_element</code> (<i>allow to iterate over the values of a table</i>).			
Keywords	characteristic of a constraint: <code>minimum</code> , <code>automaton</code> , <code>automaton without counters</code> , <code>reified automaton constraint</code> , <code>derived collection</code> . constraint network structure: <code>centered cyclic(3) constraint network(1)</code> . constraint type: <code>data constraint</code> . modelling: <code>table</code> .			

Derived Collection

$$\text{col} \left(\begin{array}{l} \text{ITEM-collection}(\text{index-dvar}, \text{value-dvar}), \\ [\text{item}(\text{index} - \text{THRESHOLD}, \text{value} - \text{VAL})] \end{array} \right)$$
Arc input(s)

ITEM TABLE

Arc generator*PRODUCT* \mapsto *collection*(item, table)**Arc arity**

2

Arc constraint(s)

- $\text{item.index} < \text{table.index}$
- $\text{item.value} = \text{table.value}$

Graph property(ies)*NARC* > 0**Sets**

SUCC \mapsto

$$\left[\begin{array}{l} \text{source}, \\ \text{variables} - \text{col} \left(\begin{array}{l} \text{VARIABLES-collection}(\text{var-dvar}), \\ [\text{item}(\text{var} - \text{TABLE.index})] \end{array} \right) \end{array} \right]$$
Constraint(s) on sets*minimum*(INDEX, variables)**Graph model**

Parts (A) and (B) of Figure 5.575 respectively show the initial and final graph associated with the second graph constraint of the **Example** slot. Since we use the *NARC* graph property, the arcs of the final graph are stressed in bold.

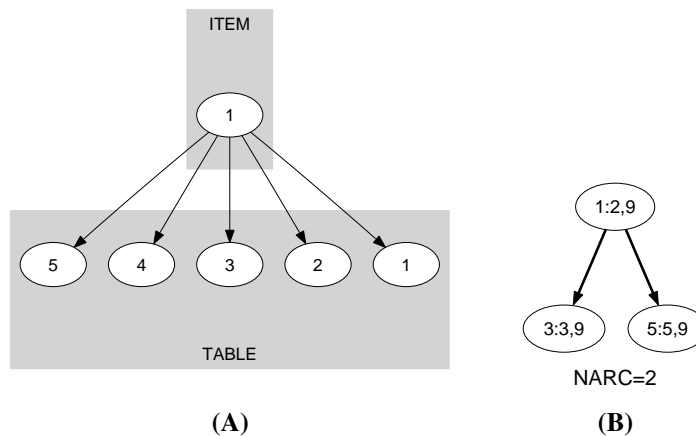


Figure 5.575: Initial and final graph of the next_element constraint

Automaton

Figure 5.576 depicts the automaton associated with the `next_element` constraint. Let I_k and V_k respectively be the `index` and the `value` attributes of the k^{th} item of the `TABLE` collections. To each quintuple $(\text{THRESHOLD}, \text{INDEX}, \text{VAL}, I_k, V_k)$ corresponds a signature variable S_k as well as the following signature constraint:

$$\begin{aligned} ((I_k \leq \text{THRESHOLD}) \wedge (I_k < \text{INDEX}) \wedge (V_k = \text{VAL})) &\Leftrightarrow S_k = 0 \wedge \\ ((I_k \leq \text{THRESHOLD}) \wedge (I_k < \text{INDEX}) \wedge (V_k \neq \text{VAL})) &\Leftrightarrow S_k = 1 \wedge \\ ((I_k \leq \text{THRESHOLD}) \wedge (I_k = \text{INDEX}) \wedge (V_k = \text{VAL})) &\Leftrightarrow S_k = 2 \wedge \\ ((I_k \leq \text{THRESHOLD}) \wedge (I_k = \text{INDEX}) \wedge (V_k \neq \text{VAL})) &\Leftrightarrow S_k = 3 \wedge \\ ((I_k \leq \text{THRESHOLD}) \wedge (I_k > \text{INDEX}) \wedge (V_k = \text{VAL})) &\Leftrightarrow S_k = 4 \wedge \\ ((I_k \leq \text{THRESHOLD}) \wedge (I_k > \text{INDEX}) \wedge (V_k \neq \text{VAL})) &\Leftrightarrow S_k = 5 \wedge \\ ((I_k > \text{THRESHOLD}) \wedge (I_k < \text{INDEX}) \wedge (V_k = \text{VAL})) &\Leftrightarrow S_k = 6 \wedge \\ ((I_k > \text{THRESHOLD}) \wedge (I_k < \text{INDEX}) \wedge (V_k \neq \text{VAL})) &\Leftrightarrow S_k = 7 \wedge \\ ((I_k > \text{THRESHOLD}) \wedge (I_k = \text{INDEX}) \wedge (V_k = \text{VAL})) &\Leftrightarrow S_k = 8 \wedge \\ ((I_k > \text{THRESHOLD}) \wedge (I_k = \text{INDEX}) \wedge (V_k \neq \text{VAL})) &\Leftrightarrow S_k = 9 \wedge \\ ((I_k > \text{THRESHOLD}) \wedge (I_k > \text{INDEX}) \wedge (V_k = \text{VAL})) &\Leftrightarrow S_k = 10 \wedge \\ ((I_k > \text{THRESHOLD}) \wedge (I_k > \text{INDEX}) \wedge (V_k \neq \text{VAL})) &\Leftrightarrow S_k = 11. \end{aligned}$$

The automaton is constructed in order to fulfil the following conditions:

- We look for an item of the `TABLE` collection such that $\text{INDEX}_i > \text{THRESHOLD}$ and $\text{INDEX}_i = \text{INDEX}$ and $\text{VALUE}_i = \text{VAL}$,
- There should not exist any item of the `TABLE` collection such that $\text{INDEX}_i > \text{THRESHOLD}$ and $\text{INDEX}_i < \text{INDEX}$ and $\text{VALUE}_i = \text{VAL}$.

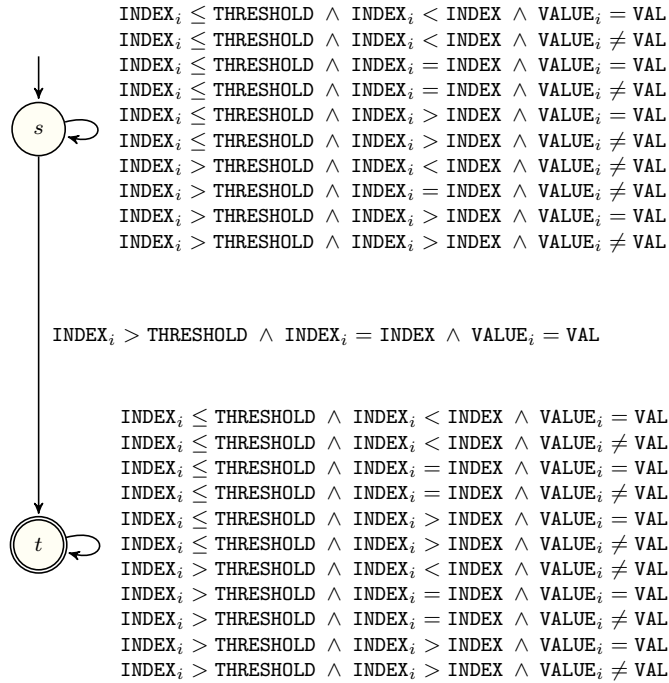


Figure 5.576: Automaton of the next_element constraint

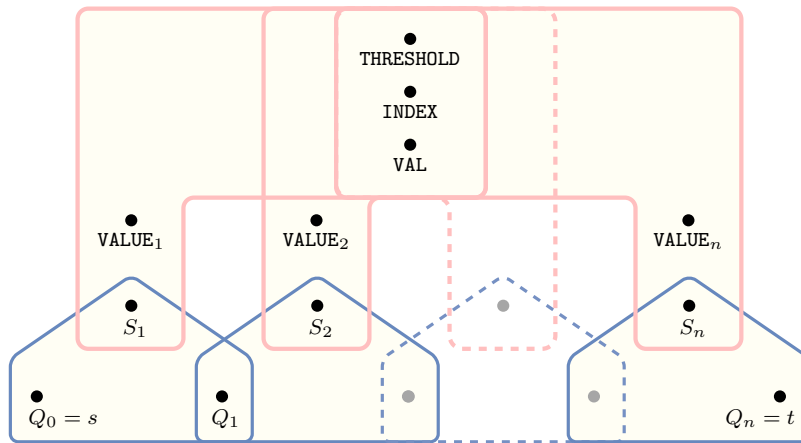


Figure 5.577: Hypergraph of the reformulation corresponding to the automaton of the next_element constraint