5.280 non_overlap_sboxes

DESCRIPTION LINKS LOGIC

Origin Geometry, derived from [38]

Constraint non_overlap_sboxes(K, DIMS, OBJECTS, SBOXES)

Synonyms non_overlap, non_overlapping.

Types

```
VARIABLES  :  collection(v−dvar)
INTEGERS   :  collection(v−int)
POSITIVES  :  collection(v−int)
```

Arguments

```
K         :  int
DIMS      :  sint
OBJECTS   :  collection(oid−int, sid−dvar, x − VARIABLES)
SBOXES    :  collection(sid−int, t − INTEGERS, l − POSITIVES)
```

Restrictions

```
|VARIABLES| ≥ 1
|INTEGERS| ≥ 1
|POSITIVES| ≥ 1
required(VARIABLES, v)
|VARIABLES| = K
required(INTEGERS, v)
|INTEGERS| = K
required(POSITIVES, v)
|POSITIVES| = K
POSITIVES.v > 0
K > 0
DIMS ≥ 0
DIMS < K
increasing_seq(OBJECTS, [oid])
required(OBJECTS, [oid, sid, x])
OBJECTS.oid ≥ 1
OBJECTS.oid ≤ |OBJECTS|
OBJECTS.sid ≥ 1
OBJECTS.sid ≤ |SBOXES|
required(SBOXES, [sid, t, l])
SBOXES.sid ≥ 1
SBOXES.sid ≤ |SBOXES|
```

Purpose

Holds if, for each pair of objects (O_i, O_j), $i < j$, O_i and O_j do not overlap with respect to a set of dimensions depicted by `DIMS`. O_i and O_j are objects that take a shape among a set of shapes. Each *shape* is defined as a finite set of shifted boxes, where each shifted box is described by a box in a `K`-dimensional space at a given offset (from the origin of the shape) with given sizes. More precisely, a *shifted box* is an entity defined by its shape id `sid`, shift offset `t`, and sizes `l`. Then, a shape is defined as the union of shifted boxes sharing the same shape id. An *object* is an entity defined by its unique object identifier `oid`, shape id `sid` and origin `x`.
An object O_i *does not overlap* an object O_j with respect to a set of dimensions depicted by `DIMS` if and only if, for all shifted box s_i associated with O_i and for all shifted box s_j associated with O_j, there exists a dimension $d \in$ `DIMS` such that the start of s_i in dimension d is greater than or equal to the end of s_j in dimension d, or the start of s_j in dimension d is greater than or equal to the end of s_i in dimension d.

Example

$$\left\langle 2, \{0,1\}, \left\langle \begin{array}{lll} \texttt{oid}-1 & \texttt{sid}-1 & \texttt{x}-\langle 4,1\rangle, \\ \texttt{oid}-2 & \texttt{sid}-3 & \texttt{x}-\langle 2,2\rangle, \\ \texttt{oid}-3 & \texttt{sid}-4 & \texttt{x}-\langle 5,4\rangle \end{array} \right\rangle, \left\langle \begin{array}{lll} \texttt{sid}-1 & \texttt{t}-\langle 0,0\rangle & \texttt{l}-\langle 1,1\rangle, \\ \texttt{sid}-1 & \texttt{t}-\langle 1,0\rangle & \texttt{l}-\langle 1,3\rangle, \\ \texttt{sid}-1 & \texttt{t}-\langle 0,2\rangle & \texttt{l}-\langle 1,1\rangle, \\ \texttt{sid}-2 & \texttt{t}-\langle 0,0\rangle & \texttt{l}-\langle 3,1\rangle, \\ \texttt{sid}-2 & \texttt{t}-\langle 0,1\rangle & \texttt{l}-\langle 1,1\rangle, \\ \texttt{sid}-2 & \texttt{t}-\langle 2,1\rangle & \texttt{l}-\langle 1,1\rangle, \\ \texttt{sid}-3 & \texttt{t}-\langle 0,0\rangle & \texttt{l}-\langle 1,2\rangle, \\ \texttt{sid}-4 & \texttt{t}-\langle 0,0\rangle & \texttt{l}-\langle 1,1\rangle \end{array} \right\rangle \right\rangle$$

Figure 5.586 shows the objects of the example. Since O_1 and O_2 do not overlap, since O_1 and O_3 do not overlap, and since O_2 and O_3 also do not overlap, the `non_overlap_sboxes` constraint holds.

Typical

$|\texttt{OBJECTS}| > 1$

Symmetries

- Items of `OBJECTS` are permutable.
- Items of `SBOXES` are permutable.
- Items of `OBJECTS.x`, `SBOXES.t` and `SBOXES.l` are permutable (*same permutation used*).
- `SBOXES.l.v` can be decreased to any value ≥ 1.

Arg. properties

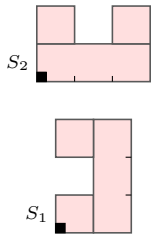
Suffix-contractible wrt. `OBJECTS`.

Remark

In addition from preventing objects to overlap, the `disjoint_sboxes` constraint also enforces that borders and corners of objects are not directly in contact.

See also

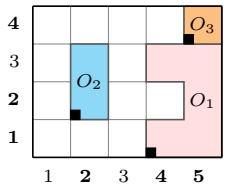
common keyword: `contains_sboxes`, `coveredby_sboxes`, `covers_sboxes` (*geometrical constraint between shifted boxes*), `diffn` (*geometrical constraint,non-overlapping*), `disjoint_sboxes`,



(A) Shapes of the first object

(B) Shape of the second object

(C) Shape of the third object



OBJECTS

O_1:	oid − 1	sid − 1	x − ⟨4, 1⟩
O_2:	oid − 2	sid − 3	x − ⟨2, 2⟩
O_3:	oid − 3	sid − 4	x − ⟨5, 4⟩

(D) Three objects where O_1 does neither overlap O_2 nor O_3 and where O_2 and O_3 also do not overlap

Figure 5.586: (D) the three pairwise non-overlapping objects O_1, O_2, O_3 of the **Example** slot respectively assigned shapes S_1, S_3, S_4; (A), (B), (C) shapes S_1, S_2, S_3 and S_4 are respectively made up from 3, 3, 1 and 1 disjoint shifted box.

`equal_sboxes` (*geometrical constraint between shifted boxes*), `geost`, `geost_time` (*geometrical constraint*,*non-overlapping*), `inside_sboxes`, `meet_sboxes`, `overlap_sboxes` (*geometrical constraint between shifted boxes*), `visible` (*geometrical constraint*).

implied by: `disjoint_sboxes`.

Keywords

constraint type: logic.

geometry: geometrical constraint, non-overlapping.

Logic

- $\texttt{origin}(\texttt{O1}, \texttt{S1}, \texttt{D}) \stackrel{\text{def}}{=} \texttt{O1.x}(\texttt{D}) + \texttt{S1.t}(\texttt{D})$
- $\texttt{end}(\texttt{O1}, \texttt{S1}, \texttt{D}) \stackrel{\text{def}}{=} \texttt{O1.x}(\texttt{D}) + \texttt{S1.t}(\texttt{D}) + \texttt{S1.l}(\texttt{D})$
- $\texttt{non\_overlap\_sboxes}(\texttt{Dims}, \texttt{O1}, \texttt{S1}, \texttt{O2}, \texttt{S2}) \stackrel{\text{def}}{=}$
  $\exists \texttt{D} \in \texttt{Dims}$
  $$\bigvee \left(\begin{array}{l} \texttt{end}(\texttt{O1}, \texttt{S1}, \texttt{D}) \leq \\ \quad \texttt{origin} \left(\begin{array}{l} \texttt{O2}, \\ \texttt{S2}, \\ \texttt{D} \end{array} \right) \\ , \\ \texttt{end}(\texttt{O2}, \texttt{S2}, \texttt{D}) \leq \\ \quad \texttt{origin} \left(\begin{array}{l} \texttt{O1}, \\ \texttt{S1}, \\ \texttt{D} \end{array} \right) \end{array} \right)$$
- $\texttt{non\_overlap\_objects}(\texttt{Dims}, \texttt{O1}, \texttt{O2}) \stackrel{\text{def}}{=}$
  $\forall \texttt{S1} \in \texttt{sboxes}([\texttt{O1.sid}])$
  $\forall \texttt{S2} \in \texttt{sboxes} \left(\left[\texttt{O2.sid} \right] \right)$
  $$\texttt{non\_overlap\_sboxes} \left(\begin{array}{l} \texttt{Dims}, \\ \texttt{O1}, \\ \texttt{S1}, \\ \texttt{O2}, \\ \texttt{S2} \end{array} \right)$$
- $\texttt{all\_non\_overlap}(\texttt{Dims}, \texttt{OIDS}) \stackrel{\text{def}}{=}$
  $\forall \texttt{O1} \in \texttt{objects}(\texttt{OIDS})$
  $\forall \texttt{O2} \in \texttt{objects} \left(\texttt{OIDS} \right)$
  $\texttt{O1.oid} < \Rightarrow$
  $\texttt{O2.oid}$
  $$\texttt{non\_overlap\_objects} \left(\begin{array}{l} \texttt{Dims}, \\ \texttt{O1}, \\ \texttt{O2} \end{array} \right)$$
- $\texttt{all\_non\_overlap}(\texttt{DIMENSIONS}, \texttt{OIDS})$