5.329 proper_forest

DESCRIPTION LINKS GRAPH

Origin

Derived from tree, [46].

Constraint

proper_forest(NTREES, NODES)

Arguments

NTREES : dvar
NODES : collection(index−int, neighbour−svar)

Restrictions

NTREES ≥ 0
required(NODES, [index, neighbour])
$|\text{NODES}| \bmod 2 = 0$
NODES.index ≥ 1
NODES.index $\leq |\text{NODES}|$
distinct(NODES, index)
NODES.neighbour ≥ 1
NODES.neighbour $\leq |\text{NODES}|$
NODES.neighbour $\neq$ NODES.index

Purpose

Cover an undirected graph G by a set of NTREES trees (i.e., a *tree* is a connected graph without cycles that contains at least two vertices [105]) in such a way that each vertex of G belongs to one distinct tree.

Example

$$\left(3, \left\langle\begin{array}{ll} \text{index}-1 & \text{neighbour}-\{3,6\}, \\ \text{index}-2 & \text{neighbour}-\{9\}, \\ \text{index}-3 & \text{neighbour}-\{1,5,7\}, \\ \text{index}-4 & \text{neighbour}-\{9\}, \\ \text{index}-5 & \text{neighbour}-\{3\}, \\ \text{index}-6 & \text{neighbour}-\{1\}, \\ \text{index}-7 & \text{neighbour}-\{3\}, \\ \text{index}-8 & \text{neighbour}-\{10\}, \\ \text{index}-9 & \text{neighbour}-\{2,4\}, \\ \text{index}-10 & \text{neighbour}-\{8\} \end{array}\right\rangle\right)$$

The proper_forest constraint holds since the undirected graph associated with the items of the NODES collection corresponds to a forest containing NTREES $= 3$ trees: each tree respectively involves the vertices $\{1, 3, 5, 6, 7\}$, $\{2, 4, 9\}$ and $\{8, 10\}$.

Typical

NTREES > 0
$|\text{NODES}| > 1$

Symmetry

Items of NODES are permutable.

Arg. properties

Functional dependency: NTREES determined by NODES.

Algorithm A filtering algorithm for the `proper_forest` constraint was proposed by N. Beldiceanu *et al.* in [46]. It achieves hybrid-consistency and its running time is dominated by the complexity of finding all edges that do not belong to any maximum cardinality matching in an undirected n-vertex, m-edge graph, i.e., $O(m \cdot n)$.

Systems `tree` in **Choco**.

See also **common keyword:** `tree` *(connected component,tree)*.

Keywords **characteristic of a constraint:** undirected graph.

constraint arguments: constraint involving set variables.

constraint type: graph constraint.

filtering: hybrid-consistency.

final graph structure: connected component, tree, no cycle, symmetric.

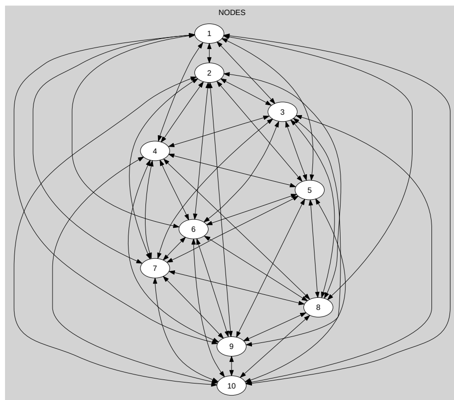
modelling: functional dependency.

Arc input(s)	NODES
Arc generator	$CLIQUE(\neq) \mapsto$collection(nodes1, nodes2)
Arc arity	2
Arc constraint(s)	in_set(nodes2.index, nodes1.neighbour)
Graph property(ies)	• **NVERTEX**= (**NARC**+2 ∗ NTREES)/2 • **NCC**= NTREES • **NVERTEX**= |NODES|
Graph class	SYMMETRIC

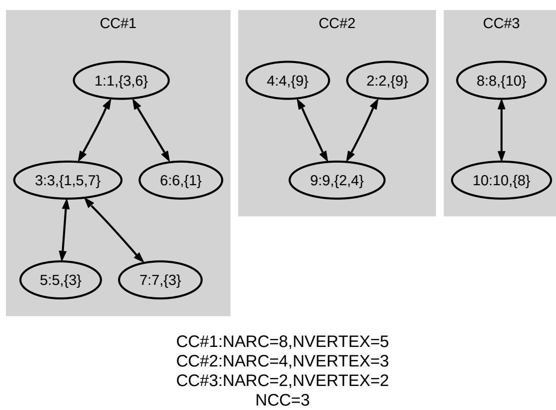
Graph model The graph constraint forces the following conditions:

- Each connected component of the final graph has n vertices and $2\cdot(n-1)$ arcs. This is equivalent to the fact that each connected component has not any cycle.
- Since we use the $CLIQUE(\neq)$ arc-generator and since, by definition, the final graph does not contain any isolated vertex, each connected component of the final graph involves more than one vertex.
- The number of connected components of the final graph is equal to **NCC**.
- All the vertices of the initial graph belong to the final graph.
- The final graph is symmetric.

Parts (A) and (B) of Figure 5.658 respectively show the initial and final graph associated with the **Example** slot. For each connected component we display its number of arcs as well as its number of vertices. The proper_forest constraint holds since the final graph has NTREES $=$ **NCC** $= 3$ connected components and no cycle.



(A)



(B)

Figure 5.658: Initial and final graph of the `proper_forest` constraint