

## 5.361 `soft_cumulative`

|                     | DESCRIPTION   | LINKS      |  |   |            |  |       |   |     |  |                    |   |     |  |                |   |      |  |
|---------------------|---|------------|--|---|------------|--|-------|---|-----|--|--------------------|---|-----|--|----------------|---|------|--|
| <b>Origin</b>       | Derived from <code>cumulative</code>  |            |  |   |            |  |       |   |     |  |                    |   |     |  |                |   |      |  |
| <b>Constraint</b>   | <code>soft_cumulative(TASKS, LIMIT, INTERMEDIATE_LEVEL, SURFACE_ON_TOP)</code>  |            |  |   |            |  |       |   |     |  |                    |   |     |  |                |   |      |  |
| <b>Arguments</b>    | <table border="0"> <tr> <td style="vertical-align: top;">TASKS</td> <td style="vertical-align: top;">:</td> <td style="vertical-align: top;">collection</td> <td style="vertical-align: middle;"> <math display="block">\left( \begin{array}{l} \text{origin-dvar,} \\ \text{duration-dvar,} \\ \text{end-dvar,} \\ \text{height-dvar} \end{array} \right)</math> </td> </tr> <tr> <td>LIMIT</td> <td>:</td> <td>int</td> <td></td> </tr> <tr> <td>INTERMEDIATE_LEVEL</td> <td>:</td> <td>int</td> <td></td> </tr> <tr> <td>SURFACE_ON_TOP</td> <td>:</td> <td>dvar</td> <td></td> </tr> </table>   |            | TASKS  | : | collection | $\left( \begin{array}{l} \text{origin-dvar,} \\ \text{duration-dvar,} \\ \text{end-dvar,} \\ \text{height-dvar} \end{array} \right)$ | LIMIT | : | int |  | INTERMEDIATE_LEVEL | : | int |  | SURFACE_ON_TOP | : | dvar |  |
| TASKS               | :   | collection | $\left( \begin{array}{l} \text{origin-dvar,} \\ \text{duration-dvar,} \\ \text{end-dvar,} \\ \text{height-dvar} \end{array} \right)$ |   |            |  |       |   |     |  |                    |   |     |  |                |   |      |  |
| LIMIT               | :   | int        |  |   |            |  |       |   |     |  |                    |   |     |  |                |   |      |  |
| INTERMEDIATE_LEVEL  | :   | int        |  |   |            |  |       |   |     |  |                    |   |     |  |                |   |      |  |
| SURFACE_ON_TOP      | :   | dvar       |  |   |            |  |       |   |     |  |                    |   |     |  |                |   |      |  |
| <b>Restrictions</b> | <pre> require_at_least(2, TASKS, [origin, duration, end]) required(TASKS, height) TASKS.duration ≥ 0 TASKS.origin ≤ TASKS.end TASKS.height ≥ 0 LIMIT ≥ 0 INTERMEDIATE_LEVEL ≥ 0 INTERMEDIATE_LEVEL ≤ LIMIT SURFACE_ON_TOP ≥ 0 </pre>  |            |  |   |            |  |       |   |     |  |                    |   |     |  |                |   |      |  |
| <b>Purpose</b>      | <p>Consider a set <math>\mathcal{T}</math> of <math>n</math> tasks described by the TASKS collection, where <math>\text{origin}_j</math>, <math>\text{duration}_j</math>, <math>\text{end}_j</math>, <math>\text{height}_j</math> are shortcuts for <math>\text{TASKS}[j].\text{origin}</math>, <math>\text{TASKS}[j].\text{duration}</math>, <math>\text{TASKS}[j].\text{end}</math>, <math>\text{TASKS}[j].\text{height}</math>. In addition let <math>\alpha</math> and <math>\beta</math> respectively denote the earliest possible start over all tasks and the latest possible end over all tasks. The <code>soft_cumulative</code> constraint forces the three following conditions:</p> <ol style="list-style-type: none"> <li>1. For each task <math>\text{TASKS}[j]</math> (<math>1 \leq j \leq n</math>) of <math>\mathcal{T}</math> we have <math>\text{origin}_j + \text{duration}_j = \text{end}_j</math>.</li> <li>2. At each point in time, the cumulated height of the set of tasks that overlap that point, does not exceed a given limit LIMIT (i.e., <math>\forall i \in [\alpha, \beta] : \sum_{j \in [1, n]   \text{origin}_j \leq i &lt; \text{end}_j} \text{height}_j \leq \text{LIMIT}</math>).</li> <li>3. The surface of the profile resource utilisation, which is greater than INTERMEDIATE_LEVEL, is equal to SURFACE_ON_TOP (i.e., <math>\sum_{i \in [\alpha, \beta]} \max(0, (\sum_{j \in [1, n]   \text{origin}_j \leq i &lt; \text{end}_j} \text{height}_j) - \text{INTERMEDIATE\_LEVEL}) = \text{SURFACE\_ON\_TOP}</math>).</li> </ol> |            |  |   |            |  |       |   |     |  |                    |   |     |  |                |   |      |  |
| <b>Example</b>      | $\left( \left\langle \begin{array}{llll} \text{origin} - 1 & \text{duration} - 4 & \text{end} - 5 & \text{height} - 1, \\ \text{origin} - 1 & \text{duration} - 1 & \text{end} - 2 & \text{height} - 2, \\ \text{origin} - 3 & \text{duration} - 3 & \text{end} - 6 & \text{height} - 2 \end{array} \right\rangle, 3, 2, 3 \right)$   |            |  |   |            |  |       |   |     |  |                    |   |     |  |                |   |      |  |

Figure 5.703 shows the cumulated profile associated with the example. To each

task of the `cumulative` constraint corresponds a set of rectangles coloured with the same colour: the sum of the lengths of the rectangles corresponds to the duration of the task, while the height of the rectangles (i.e., all the rectangles associated with a task have the same height) corresponds to the resource consumption of the task. The `soft_cumulative` constraint holds since:

1. For each task we have that its end is equal to the sum of its origin and its duration.
2. At each point in time we do not have a cumulated resource consumption strictly greater than the upper limit  $LIMIT = 3$  enforced by the second argument of the `soft_cumulative` constraint.
3. The surface of the cumulated profile located on top of the intermediate level  $INTERMEDIATE\_LEVEL = 2$  is equal to  $SURFACE\_ON\_TOP = 3$ .

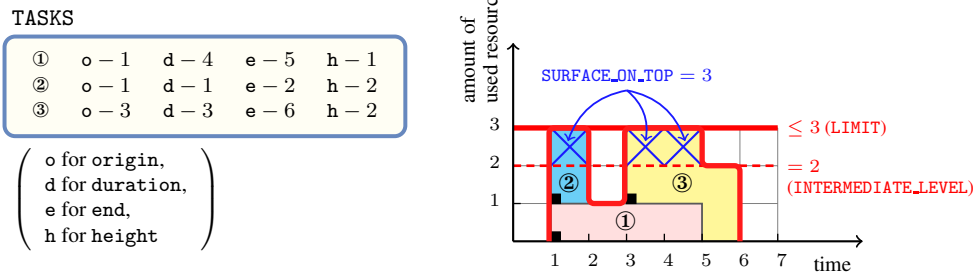


Figure 5.703: Resource consumption profile associated with the three tasks of the **Example** slot, where parts on top of the intermediate level 2 are marked by a cross

### Typical

```

|TASKS| > 1
range(TASKS.origin) > 1
range(TASKS.duration) > 1
range(TASKS.end) > 1
range(TASKS.height) > 1
TASKS.duration > 0
TASKS.height > 0
LIMIT < sum(TASKS.height)
INTERMEDIATE_LEVEL > 0
INTERMEDIATE_LEVEL < LIMIT
SURFACE_ON_TOP > 0

```

### Symmetries

- Items of `TASKS` are [permutable](#).
- One and the same constant can be [added](#) to the origin and end attributes of all items of `TASKS`.
- `LIMIT` can be [increased](#).

### Remark

The `soft_cumulative` constraint was initially introduced in [CHIP](#) [124] as a variant of the `cumulative` constraint. An extension of this constraint where one can restrict the surface on top of the intermediate level on different time intervals was first proposed in [311] and was generalised in [118].

**See also**

**hard version:** [cumulative](#).

**Keywords**

**constraint type:** [predefined constraint](#), [soft constraint](#), [scheduling constraint](#), [resource constraint](#), [temporal constraint](#), [relaxation](#).

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