5.381 strongly_connected

DESCRIPTION LINKS GRAPH

Origin [5]

Constraint strongly_connected(NODES)

Argument NODES : collection(index−int, succ−svar)

Restrictions
required(NODES, [index, succ])
NODES.index ≥ 1
NODES.index ≤ |NODES|
distinct(NODES, index)

Purpose Consider a digraph G described by the NODES collection. Select a subset of arcs of G so that we have a single strongly connected component involving all vertices of G.

Example

$$\left\{ \left\langle \begin{array}{ll} \texttt{index} - 1 & \texttt{succ} - \{2\}, \\ \texttt{index} - 2 & \texttt{succ} - \{3\}, \\ \texttt{index} - 3 & \texttt{succ} - \{2, 5\}, \\ \texttt{index} - 4 & \texttt{succ} - \{1\}, \\ \texttt{index} - 5 & \texttt{succ} - \{4\} \end{array} \right\rangle \right\}$$

The strongly_connected constraint holds since the NODES collection depicts a graph involving a single strongly connected component (i.e., since we have a circuit visiting successively the vertices 1, 2, 3, 5, and 4).

Typical |NODES| > 2

Symmetry Items of NODES are permutable.

Algorithm The sketch of a filtering algorithm for the strongly_connected constraint is given in [142, page 89].

See also
common keyword: link_set_to_booleans *(constraint involving set variables)*.
implied by: connected.
related: circuit *(one single strongly connected component in the final solution)*.

Keywords
constraint arguments: constraint involving set variables.
constraint type: graph constraint.
filtering: linear programming.
final graph structure: strongly connected component.

Arc input(s)	NODES
Arc generator	$CLIQUE \mapsto$ collection(nodes1, nodes2)
Arc arity	2
Arc constraint(s)	in_set(nodes2.index, nodes1.succ)
Graph property(ies)	MIN_NSCC= |NODES|

Graph model Part (A) of Figure 5.742 shows the initial graph from which we start. It is derived from the set associated with each vertex. Each set describes the potential values of the succ attribute of a given vertex. Part (B) of Figure 5.742 gives the final graph associated with the **Example** slot. The strongly_connected constraint holds since the final graph contains a single strongly connected component mentioning every vertex of the initial graph.

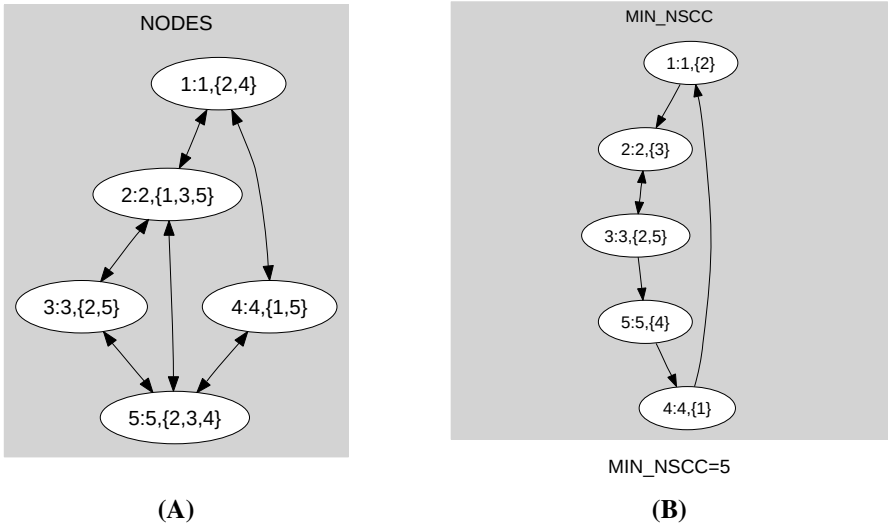

Figure 5.742: Initial and final graph of the strongly_connected set constraint

Signature Since the maximum number of vertices of the final graph is equal to |NODES| we can rewrite the graph property $\mathbf{MIN\_NSCC} = |\texttt{NODES}|$ to $\mathbf{MIN\_NSCC} \geq |\texttt{NODES}|$ and simplify $\underline{\mathbf{MIN\_NSCC}}$ to $\mathbf{MIN\_NSCC}$.