

## 5.402 track

|              | DESCRIPTION   | LINKS | GRAPH |
|--------------|---|-------|-------|
| Origin       | [274]   |       |       |
| Constraint   | track(NTRAIL, TASKS)  |       |       |
| Arguments    | NTRAIL : int<br>TASKS : collection(trail-int, origin-dvar, end-dvar)  |       |       |
| Restrictions | NTRAIL > 0<br>NTRAIL ≤  TASKS <br> TASKS  > 0<br>required(TASKS, [trail, origin, end])<br>TASKS.origin ≤ TASKS.end  |       |       |
| Purpose      | <div style="border: 1px solid pink; padding: 5px;">           The track constraint forces that, at each point in time overlapped by at least one task, the number of distinct values of the trail attribute of the set of tasks that overlap that point, is equal to NTRAIL.         </div>   |       |       |
| Example      | <div style="border: 1px solid blue; padding: 10px; display: inline-block;"> <math display="block">\left( 2, \left\langle \begin{array}{lll} \text{trail} - 1 &amp; \text{origin} - 1 &amp; \text{end} - 2, \\ \text{trail} - 2 &amp; \text{origin} - 1 &amp; \text{end} - 2, \\ \text{trail} - 1 &amp; \text{origin} - 2 &amp; \text{end} - 4, \\ \text{trail} - 2 &amp; \text{origin} - 2 &amp; \text{end} - 3, \\ \text{trail} - 2 &amp; \text{origin} - 3 &amp; \text{end} - 4 \end{array} \right\rangle \right)</math> </div>                                     |       |       |
|              | Figure 5.761 represents the tasks of the example: to the $i^{\text{th}}$ task of the TASKS collection corresponds a rectangle labelled by $i$ . The track constraint holds since:   |       |       |
|              | <ul style="list-style-type: none"> <li>• The first and second tasks both overlap instant 1 and have a respective trail of 1 and 2. This makes two distinct values for the trail attribute at instant 1.</li> <li>• The third and fourth tasks both overlap instant 2 and have a respective trail of 1 and 2. This makes two distinct values for the trail attribute at instant 2.</li> <li>• The third and fifth tasks both overlap instant 3 and have a respective trail of 1 and 2. This makes two distinct values for the trail attribute at instant 3.</li> </ul> |       |       |
| Typical      | NTRAIL <  TASKS <br> TASKS  > 1<br>range(TASKS.trail) > 1<br>TASKS.origin < TASKS.end   |       |       |
| Symmetries   | <ul style="list-style-type: none"> <li>• Items of TASKS are <b>permutable</b>.</li> <li>• All occurrences of two distinct values of TASKS.trail can be <b>swapped</b>; all occurrences of a value of TASKS.trail can be <b>renamed</b> to any unused value.</li> <li>• One and the same constant can be <b>added</b> to the origin and end attributes of all items of TASKS.</li> </ul>   |       |       |

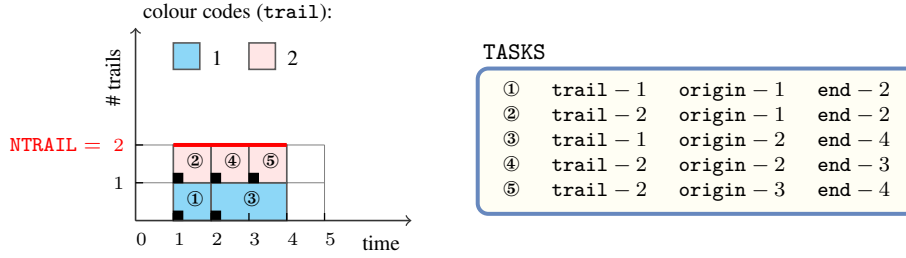


Figure 5.761: The tasks associated with the example of the **Example** slot, at each instant we have two distinct values for the trail attribute ( $NTRAIL = 2$ )

### Reformulation

The track constraint can be expressed in term of a set of reified constraints and of  $2 \cdot |\text{TASKS}|$  `nvalue` constraints:

- For each pair of tasks  $\text{TASKS}[i], \text{TASKS}[j]$  ( $i, j \in [1, |\text{TASKS}|]$ ) of the **TASKS** collection we create a variable  $T_{ij}^{\text{origin}}$  which is set to the **trail** attribute of task  $\text{TASKS}[j]$  if task  $\text{TASKS}[j]$  overlaps the origin attribute of task  $\text{TASKS}[i]$ , and to the **trail** attribute of task  $\text{TASKS}[i]$  otherwise:
  - If  $i = j$ :
    - $T_{ij}^{\text{origin}} = \text{TASKS}[i].\text{trail}$ .
  - If  $i \neq j$ :
    - $T_{ij}^{\text{origin}} = \text{TASKS}[i].\text{trail} \vee T_{ij}^{\text{origin}} = \text{TASKS}[j].\text{trail}$ .
    - $((\text{TASKS}[j].\text{origin} \leq \text{TASKS}[i].\text{origin} \wedge \text{TASKS}[j].\text{end} > \text{TASKS}[i].\text{origin}) \wedge (T_{ij}^{\text{origin}} = \text{TASKS}[j].\text{trail})) \vee ((\text{TASKS}[j].\text{origin} > \text{TASKS}[i].\text{origin} \vee \text{TASKS}[j].\text{end} \leq \text{TASKS}[i].\text{origin}) \wedge (T_{ij}^{\text{origin}} = \text{TASKS}[i].\text{trail}))$
- For each task  $\text{TASKS}[i]$  ( $i \in [1, |\text{TASKS}|]$ ) we impose the number of distinct trails associated with the tasks that overlap the origin of task  $\text{TASKS}[i]$  ( $\text{TASKS}[i]$  overlaps its own origin) to be equal to  $NTRAIL$ :
  - `nvalue`( $NTRAIL, \langle T_{i1}^{\text{origin}}, T_{i2}^{\text{origin}}, \dots, T_{i|\text{TASKS}|}^{\text{origin}} \rangle$ ).
- For each pair of tasks  $\text{TASKS}[i], \text{TASKS}[j]$  ( $i, j \in [1, |\text{TASKS}|]$ ) of the **TASKS** collection we create a variable  $T_{ij}^{\text{end}}$  which is set to the **trail** attribute of task  $\text{TASKS}[j]$  if task  $\text{TASKS}[j]$  overlaps the end attribute of task  $\text{TASKS}[i]$ , and to the **trail** attribute of task  $\text{TASKS}[i]$  otherwise:
  - If  $i = j$ :
    - $T_{ij}^{\text{end}} = \text{TASKS}[i].\text{trail}$ .
  - If  $i \neq j$ :
    - $T_{ij}^{\text{end}} = \text{TASKS}[i].\text{trail} \vee T_{ij}^{\text{end}} = \text{TASKS}[j].\text{trail}$ .
    - $((\text{TASKS}[j].\text{origin} \leq \text{TASKS}[i].\text{end} - 1 \wedge \text{TASKS}[j].\text{end} > \text{TASKS}[i].\text{end} - 1) \wedge (T_{ij}^{\text{end}} = \text{TASKS}[j].\text{trail})) \vee ((\text{TASKS}[j].\text{origin} > \text{TASKS}[i].\text{end} - 1 \vee \text{TASKS}[j].\text{end} \leq \text{TASKS}[i].\text{end} - 1) \wedge (T_{ij}^{\text{end}} = \text{TASKS}[i].\text{trail}))$
- For each task  $\text{TASKS}[i]$  ( $i \in [1, |\text{TASKS}|]$ ) we impose the number of distinct trails associated with the tasks that overlap the end of task  $\text{TASKS}[i]$  ( $\text{TASKS}[i]$  overlaps its

own end) to be equal to NTRAIL:

$\text{nvalue}(\text{NTRAIL}, \langle T_{i1}^{\text{end}}, T_{i2}^{\text{end}}, \dots, T_{i|\text{TASKS}|}^{\text{end}} \rangle)$ .

With respect to the **Example** slot we get the following conjunction of **nvalue** constraints:

- The **nvalue**(2, ⟨1, 2, 1, 1, 1⟩) constraint corresponding to the **trail** attributes of the tasks that overlap the origin of the first task (i.e., instant 1) that has a trail of 1.
- The **nvalue**(2, ⟨1, 2, 2, 2, 2⟩) constraint corresponding to the **trail** attributes of the tasks that overlap the origin of the second task (i.e., instant 1) that has a trail of 2.
- The **nvalue**(2, ⟨1, 1, 1, 2, 1⟩) constraint corresponding to the **trail** attributes of the tasks that overlap the origin of the third task (i.e., instant 2) that has a trail of 1.
- The **nvalue**(2, ⟨2, 2, 1, 2, 2⟩) constraint corresponding to the **trail** attributes of the tasks that overlap the origin of the fourth task (i.e., instant 2) that has a trail of 2.
- The **nvalue**(2, ⟨2, 2, 1, 2, 2⟩) constraint corresponding to the **trail** attributes of the tasks that overlap the origin of the fifth task (i.e., instant 3) that has a trail of 2.
- The **nvalue**(2, ⟨1, 2, 1, 1, 1⟩) constraint corresponding to the **trail** attributes of the tasks that overlap the last instant of the first task (i.e., instant 1) that has a trail of 1.
- The **nvalue**(2, ⟨1, 2, 2, 2, 2⟩) constraint corresponding to the **trail** attributes of the tasks that overlap the last instant of the second task (i.e., instant 1) that has a trail of 2.
- The **nvalue**(2, ⟨1, 1, 1, 1, 2⟩) constraint corresponding to the **trail** attributes of the tasks that overlap the last instant of the third task (i.e., instant 3) that has a trail of 1.
- The **nvalue**(2, ⟨2, 2, 1, 2, 2⟩) constraint corresponding to the **trail** attributes of the tasks that overlap the last instant of the fourth task (i.e., instant 2) that has a trail of 2.
- The **nvalue**(2, ⟨2, 2, 1, 2, 2⟩) constraint corresponding to the **trail** attributes of the tasks that overlap the last instant of the fifth task (i.e., instant 3) that has a trail of 2.

**See also**

**common keyword:** *coloured\_cumulative* (*resource constraint*).

**implies (items to collection):** *atleast\_nvector*.

**used in graph description:** *nvalue*.

**Keywords**

**characteristic of a constraint:** *derived collection*.

**constraint type:** *timetabling constraint*, *resource constraint*, *temporal constraint*.

**Derived Collection**

$$\text{col} \left( \begin{array}{c} \text{TIME\_POINTS} - \text{collection} \left( \begin{array}{c} \text{origin} - \text{dvar}, \\ \text{end} - \text{dvar}, \\ \text{point} - \text{dvar} \end{array} \right), \\ \left[ \begin{array}{c} \text{item} \left( \begin{array}{c} \text{origin} - \text{TASKS.origin}, \\ \text{end} - \text{TASKS.end}, \\ \text{point} - \text{TASKS.origin} \end{array} \right), \\ \text{item} \left( \begin{array}{c} \text{origin} - \text{TASKS.origin}, \\ \text{end} - \text{TASKS.end}, \\ \text{point} - \text{TASKS.end} - 1 \end{array} \right) \end{array} \right] \end{array} \right)$$

**Arc input(s)**

TASKS

**Arc generator** $SELF \mapsto \text{collection}(\text{tasks})$ **Arc arity**

1

**Arc constraint(s)** $\text{tasks.origin} \leq \text{tasks.end}$ **Graph property(ies)** $\underline{\text{NARC}} = |\text{TASKS}|$ **Arc input(s)**

TIME\_POINTS TASKS

**Arc generator** $PRODUCT \mapsto \text{collection}(\text{time\_points}, \text{tasks})$ **Arc arity**

2

**Arc constraint(s)**

- $\text{time\_points.end} > \text{time\_points.origin}$
- $\text{tasks.origin} \leq \text{time\_points.point}$
- $\text{time\_points.point} < \text{tasks.end}$

**Sets**

$$\text{SUCC} \mapsto \left[ \begin{array}{c} \text{source}, \\ \text{variables} - \text{col} \left( \begin{array}{c} \text{VARIABLES} - \text{collection}(\text{var} - \text{dvar}), \\ \text{item}(\text{var} - \text{TASKS.trail}) \end{array} \right) \end{array} \right]$$

**Constraint(s) on sets** $nvalue(\text{NTRAIL}, \text{variables})$ **Graph model**

Parts (A) and (B) of Figure 5.762 respectively show the initial and final graph of the second graph constraint of the **Example** slot.

**Signature**

Consider the first graph constraint. Since we use the  $SELF$  arc generator on the TASKS collection, the maximum number of arcs of the final graph is equal to  $|\text{TASKS}|$ . Therefore we can rewrite  $\text{NARC} = |\text{TASKS}|$  to  $\text{NARC} \geq |\text{TASKS}|$  and simplify  $\underline{\text{NARC}}$  to  $\overline{\text{NARC}}$ .

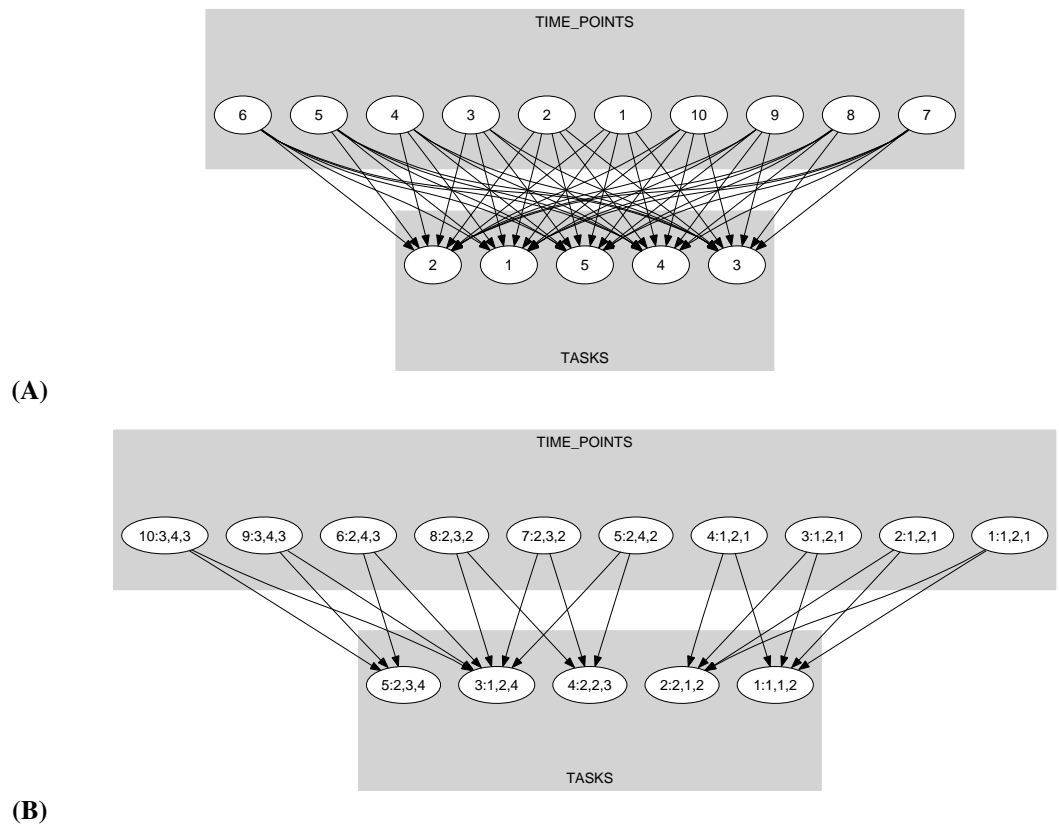


Figure 5.762: Initial and final graph of the track constraint

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