

5.407 two_layer_edge_crossing

	DESCRIPTION	LINKS	GRAPH
Origin	Inspired by [201].		
Constraint	two_layer_edge_crossing $\left(\begin{array}{l} \text{NCROSS,} \\ \text{VERTICES_LAYER1,} \\ \text{VERTICES_LAYER2,} \\ \text{EDGES} \end{array} \right)$		
Arguments	NCROSS : dvar VERTICES_LAYER1 : collection(id-int, pos-dvar) VERTICES_LAYER2 : collection(id-int, pos-dvar) EDGES : collection(id-int, vertex1-int, vertex2-int)		
Restrictions	NCROSS ≥ 0 required(VERTICES_LAYER1, [id, pos]) VERTICES_LAYER1.id ≥ 1 VERTICES_LAYER1.id $\leq \text{VERTICES_LAYER1} $ distinct(VERTICES_LAYER1, id) distinct(VERTICES_LAYER1, pos) required(VERTICES_LAYER2, [id, pos]) VERTICES_LAYER2.id ≥ 1 VERTICES_LAYER2.id $\leq \text{VERTICES_LAYER2} $ distinct(VERTICES_LAYER2, id) distinct(VERTICES_LAYER2, pos) required(EDGES, [id, vertex1, vertex2]) EDGES.id ≥ 1 EDGES.id $\leq \text{EDGES} $ distinct(EDGES, id) EDGES.vertex1 ≥ 1 EDGES.vertex1 $\leq \text{VERTICES_LAYER1} $ EDGES.vertex2 ≥ 1 EDGES.vertex2 $\leq \text{VERTICES_LAYER2} $		
Purpose	NCROSS is the number of line segments intersections.		
Example	$\left(\begin{array}{l} 2, \langle \text{id} - 1 \text{ pos} - 1, \text{id} - 2 \text{ pos} - 2 \rangle, \\ \langle \text{id} - 1 \text{ pos} - 3, \text{id} - 2 \text{ pos} - 1, \text{id} - 3 \text{ pos} - 2 \rangle, \\ \langle \text{id} - 1 \text{ vertex1} - 2 \text{ vertex2} - 2, \\ \text{id} - 2 \text{ vertex1} - 2 \text{ vertex2} - 3, \\ \text{id} - 3 \text{ vertex1} - 1 \text{ vertex2} - 1 \rangle \end{array} \right)$		

Figure 5.770 provides a picture of the example, where one can see the two line segments intersections. Each line segment of Figure 5.770 is labelled with its identifier and corresponds to an item of the EDGES collection. The two vertices on top of Figure 5.770

correspond to the items of the VERTICES_LAYER1 collection, while the three other vertices are associated with the items of VERTICES_LAYER2.

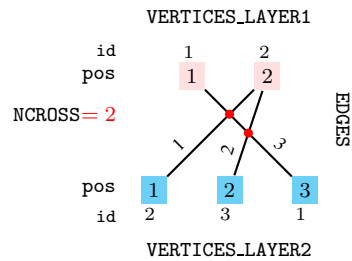


Figure 5.770: Intersection between line segments joining two layers of the **Example** slot for the constraint `two_layer_edge_crossing(NCROSS, VERTICES_LAYER1, VERTICES_LAYER2, EDGES)`

Typical

```
|VERTICES_LAYER1| > 1
|VERTICES_LAYER2| > 1
|EDGES| ≥ |VERTICES_LAYER1|
|EDGES| ≥ |VERTICES_LAYER2|
```

Symmetries

- Arguments are [permutable](#) w.r.t. permutation (NCROSS) (VERTICES_LAYER1, VERTICES_LAYER2) (EDGES).
- Items of VERTICES_LAYER1 are [permutable](#).
- Items of VERTICES_LAYER2 are [permutable](#).

Arg. properties

Functional dependency: NCROSS determined by VERTICES_LAYER1, VERTICES_LAYER2 and EDGES.

Remark

The two-layer edge crossing minimisation problem was proved to be NP-hard in [184].

See also

common keyword: [crossing](#), [graph_crossing](#) (*line segments intersection*).

Keywords

characteristic of a constraint: derived collection.
constraint arguments: pure functional dependency.
geometry: geometrical constraint, line segments intersection.
miscellaneous: obscure.
modelling: functional dependency.

Derived Collection

$$\text{col} \left(\left[\text{item} \left(\begin{array}{l} \text{layer1} - \text{EDGES.vertex1}(\text{VERTICES_LAYER1}, \text{pos}, \text{id}), \\ \text{layer2} - \text{EDGES.vertex2}(\text{VERTICES_LAYER2}, \text{pos}, \text{id}) \end{array} \right) \right] \right)$$

Arc input(s)

EDGES_EXTREMITIES

Arc generator*CLIQUE*(<) \mapsto *collection*(edges_extremities1, edges_extremities2)**Arc arity**

2

Arc constraint(s)

$$\bigvee \left(\begin{array}{l} \bigwedge \left(\begin{array}{l} \text{edges_extremities1.layer1} < \text{edges_extremities2.layer1}, \\ \text{edges_extremities1.layer2} > \text{edges_extremities2.layer2} \end{array} \right), \\ \bigwedge \left(\begin{array}{l} \text{edges_extremities1.layer1} > \text{edges_extremities2.layer1}, \\ \text{edges_extremities1.layer2} < \text{edges_extremities2.layer2} \end{array} \right) \end{array} \right)$$

Graph property(ies)NARC= NCROSS**Graph model**

As usual for the two-layer edge crossing problem [201], [22], positions of the vertices on each layer are represented as a permutation of the vertices. We generate a derived collection that, for each edge, contains the position of its extremities on both layers. In the arc generator we use the restriction < in order to generate a single arc for each pair of segments. This is required, since otherwise we would count more than once a line segments intersection.

Parts (A) and (B) of Figure 5.771 respectively show the initial and final graph associated with the **Example** slot. Since we use the NARC graph property, the arcs of the final graph are stressed in bold.

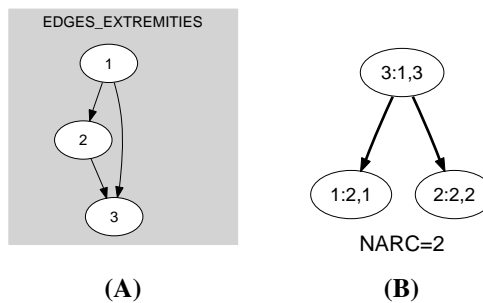


Figure 5.771: Initial and final graph of the two_layer_edge_crossing constraint

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