M2 ORO: Advanced Integer Programming
Lecture 4
Exercices of Modeling

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1 Multi-Item Lot-Sizing Problem (MLS)

Problème 1 The Multi-Item Lot-Sizing Problem (MLS).
Given demands d_t^k for items $k = 1, \dots, K$ over a time horizon $t = 1, \dots, T$. All items must be produced on a single machine. The machine has to produce exactly one item type in each period. Furthermore, the machine has no capacity : it can produce any number of items in a given period. Given unit production costs p_t^k, unit storage costs h_t^k and set-up costs f_t^k for each item k in each period t, we wish to find a minimum cost production plan.

Question 1 *Model this problem (MLS) as a Mixed Integer Linear Program.*

Now, suppose that we can compute all the feasible production plans for each item k individually, i.e. all the feasible solutions of the Uncapacitated Lot-Sizing Problem induced for each item k. Let X^k denote this set of feasible solutions and n^k the cardinality of X^k. Show that (MLS) can be seen as an instance of the Set-Partitioning Problem and derive a second model for (MLS) as a Binary Integer Linear Program with $\sum_{k=1}^{K} n^k$ binary variables.

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2 Capacitated Facility Location Problem (CFL)

Problème 2 The Capacitated Facility Location Problem (CFL).
Given a set of potential depots $J = \{1, \dots, n\}$ and a set of clients $I = \{1, \dots, m\}$, there is a number of items, all of the same type, to serve from the depots to the clients. Each depot $j \in J$ has a finite capacity b_j (the number of items available at the depot) and each client $i \in I$ has a finite requirement a_i (the minimum number of items ordered by the client). There is a fixed cost f_j associated with the use of depot $j \in J$ and an unit transportation cost c_{ij} that is paid for each item served to client $i \in I$ from depot $j \in J$. All data are positive integers. The problem is to decide which depots to open and which amount of items to serve to each client from each open depot, so as to minimize the sum of the fixed and transportation costs.

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3 Change-Making Problem

Problème 3 The Change Making Problem (CMP).
A cashier has to assemble a given change $c \in \mathbb{Z}_{*+}$ using the less number of coins of specified values $w_j \in \mathbb{Z}_{*+}$, $j = 1, \dots, n$. For each value, an unlimited number of coins is available.

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4 Traveling Salesman Problem with Time Windows (TSP-TW)

Problème 4 The Traveling Salesman Problem with Time Windows (TSP-TW).
The TSPTW is defined on a network $G = (N \cup \{0\}, A)$ *where* $N = \{1, \dots, n\}$ *is the set of nodes to visit,* 0 *is the depot, and* A *is the set of arcs connecting each pairs of distinct nodes. To each arc* $(i, j) \in A$*, are associated a cost* $c_{ij} \geqslant 0$ *and a travel duration* $t_{ij} > 0$*. To each node* $i \in N \cup \{0\}$*, is associated a time window* $[a_i, b_i]$*, with* $0 \leqslant a_i \leqslant b_i$*. A* ***tour*** *is a path starting at the depot at time* 0*, visiting all nodes in* N *exactly once, then returning to the depot. Given a tour, we can associate an* ***arrival time*** *to each node* $i \in N \cup \{0\}$ *such that : if arc* $(0, i)$ *belongs to the tour then the arrival time at* i *is greater or equal to the travel duration* t_{0i} *; and if arc* (i, j)*,* $i \neq 0$*, belongs to the tour then the arrival time at* j *is greater or equal to the arrival time at* i *plus the travel duration* t_{ij}*. If the arrival time at each node* $i \in N \cup \{0\}$ *belongs to the time interval* $[a_i, b_i]$*, then the tour and the associated arrival time vector form a* ***feasible tour****. The problem is to find the feasible tour of minimum cost.*

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5 Dominating Set

Problème 5 The Minimum Weighted Dominating Set Problem *Given a number of locations, the problem is to select some locations to install emergency service centers. For each possible location, we know the cost of installing a service center at this location, and which locations (in its direct neighborhood) it will be able to service. The goal is to choose a minimum cost set of service centers so that each location is covered.*

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6 Graph Clustering

Problème 6 The Graph Clustering Problem.
Consider a complete graph $G = (V, E)$*, a constant* $K \in \mathbb{Z}_{+*}$*, a cost* $c_e > 0$ *for each edge* $e \in E$*, a weight* $d_i \geqslant 0$ *for each node* $i \in V$*, and a cluster capacity* C *with* $\min_{i \in V} d_i \leqslant C < \sum_{i \in V} d_i$*.*

A ***capacitated cluster*** *of* G *is a (possibly empty) subset of nodes satisfying the property that the sum of the node weights does not exceed capacity* C *; an edge is included in a cluster if it joins two nodes within the cluster.*

The problem is to split node set V *into* K *capacitated clusters such that any node belongs to at most one cluster and the sum of the costs of the edges included in any clusters is maximized.*

Question 2 *Model this problem as a Binary Linear Program. Identify redundant constraints. Reformulate the problem as a set-*ing problem and derive a second Binary Linear Program formulation.*

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7 Graph Bandwidth

Problème 7 The Graph Bandwidth Problem.
Consider an undirected connected graph $G = (V, E)$ *with* $|V| = n$ *vertices and* $|E| = m$ *edges. A* linear layout *of* G *is a numbering of the vertices of* G*. In other words, it is an assignment* $f : V \to \{1, 2, \dots, n\}$*, such that different vertices* $u, v \in V$ *have different numbers* $f(u) \neq f(v)$*. The* bandwidth *of layout* f*, denoted by* $\Phi_f(G)$*, is the maximum difference between the numbers assigned to adjacent vertices, i.e.* $\Phi_f(G) = \max\{|f(u) - f(v)|, (u, v) \in E\}$*. The Graph Bandwidth Problem is to find the minimum bandwidth over all possible linear layouts of* G*. This value, denoted by* $\Phi(G)$*, is called the bandwidth of* G*.*

Question 3 Model this problem as an Integer Linear Program.

Q3.1. Compute the optimal value of its continuous relaxation.

Q3.2. Show that $d(u, v)\Phi_f(G) \geqslant f(u) - f(v)$ for all pair of vertices $u, v \in V$: where $d(u, v)$ denotes the minimum number of edges in a path from u to v.

Q3.3. Derive a family of valid inequalities for your Integer Linear Program, and show how these inequalities are stronger than the ones of your initial model.

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8 Model of logic

Problème 8 *Suppose you are interested in choosing a set of investments among seven possible investments numbered from 1 to 7. The estimate profit of each investment* $i = 1..7$ *is given by a positive integer* $c_i \in \mathbb{Z}_{*+}$. *You want to maximize your profit, knowing that :*

1. *you cannot invest in all of them*
2. *you must choose at least one of them*
3. *at most one of investments 1 and 3 can be chosen*
4. *investment 4 can be chosen only if investment 2 is also chosen*
5. *you must choose either both of investments 1 and 5, or neither*
6. *you must choose at least one of investments 1,2,3 or at least two of investments 2,4,5,6*

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9 Recycling

Problème 9 n *cities have organized their own recycling policy : they allocate different colours (red, orange, yellow, green and blue) to the bins corresponding to each given waste type (Plastic, Glass, Aluminium, Steel, Newspaper). The government wants to harmonize the colour scheme for all cities by minimizing the total changes over every cities.*