Scheduling pumps and reservoirs with integer nonlinear programming and deep learning

Sophie Demassey, Valentina Sessa, Amirhossein Tavakoli (Mines Paris – PSL)

SimHydro 2025

DECISION: PREDICTION OR PRESCRIPTION

decision aid: compute one of the best possible options

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- data/computation intensive

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- *models are based on data forecasts*

machine learning

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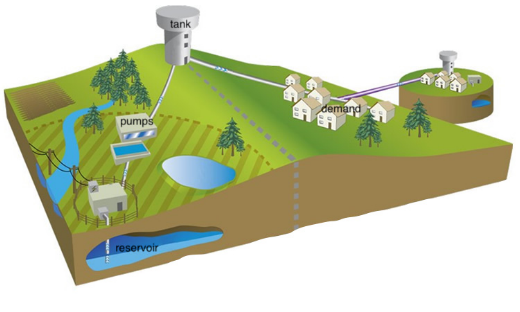
- no certificate
- data/computation intensive
- *algorithms are based on optimization*

combine MO and ML when models are complex but certificates required

LOAD SHIFTING IN DRINKING WATER DISTRIBUTION

pump in advance of demand to save energy

- water/energy storage tanks
- nonlinear efficiency
- dynamic electricity tariff



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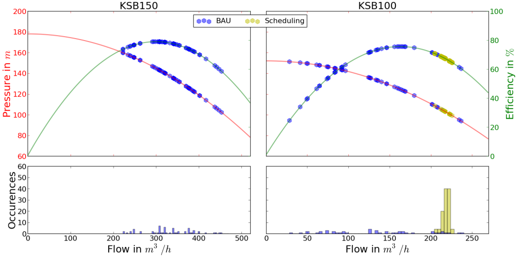

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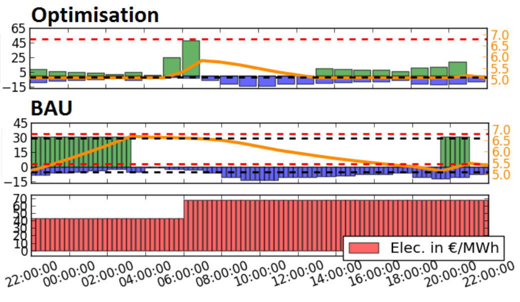

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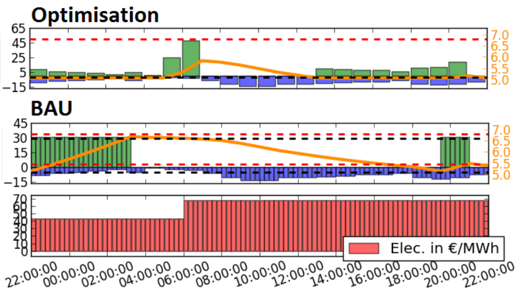
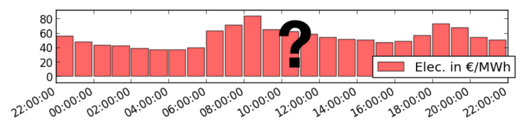
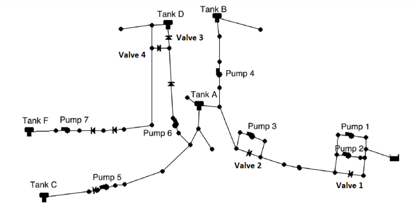


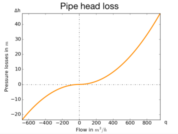


ACCURATE BUT COMPLEX ANALYTICAL MODEL



nonconvex flow/head loss equation $\Delta h = \phi(q)$

friction in pipes



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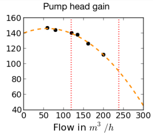

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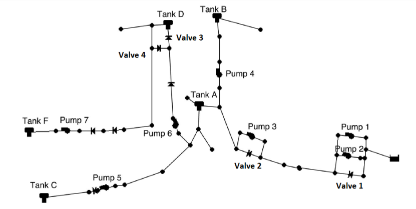

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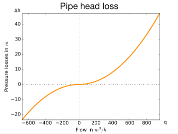

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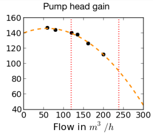

mixed integer nonconvex model

$$min \sum_t C_t \gamma_t(q_t, x_t) :$$

$$\underline{H}^R \leq h_t^R \leq \overline{H}^R \qquad \forall t$$

$$h_{t+1}^R = h_t^R + \sigma q_t^R \qquad \forall t$$

$$q_t^S = D_t^S \qquad \forall t$$

$$(\Delta h_t - \phi(q_t))^\top x_t = 0 \qquad \forall t$$

$$q_t^\top (1 - x_t) = 0 \qquad \forall t$$

on/off switch $x_{ta} \in \{0,1\}$

arc flow q_{ta} and head loss Δh_{ta}

reservoir/service node inflow q_{tr}^R, q_{ts}^S and head h_t

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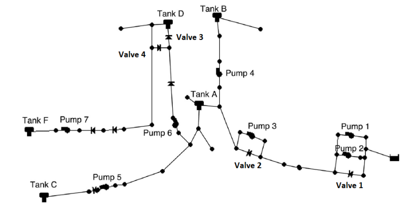

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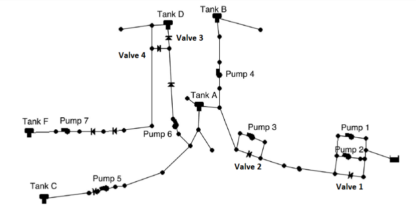

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- computing $(q_t, h_t) \in Eq(x_t, D_t^S, h_t^R)$ is easy (Todini & Pilati's Newton algorithm/EPANET)
- but optimizing $(x_t)_t$ is hard

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integer/nonconvex bilevel model

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   simplify some of the hardest parts
   - PWL approx [Morsi12,...]
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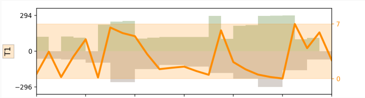
tight tank limits, long time steps



$\Rightarrow$ scarce/sparse feasibility set in discrete x-space

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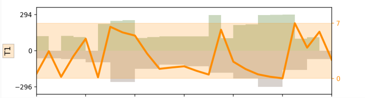

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SEARCH THE CONTINUOUS h^R-SPACE

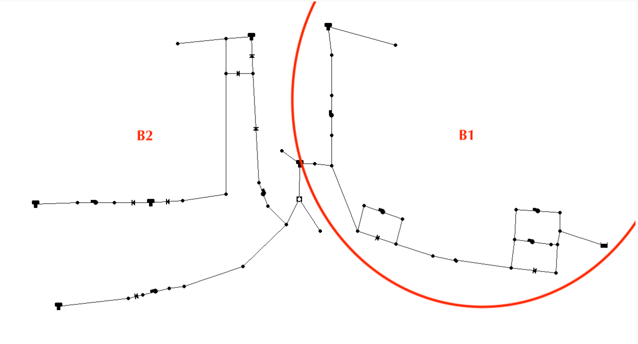
Sketch of the algorithm

1. fix the tank level profiles h^R
2. compute all equilibria $(q_t, h_t) \in Eq(x_t, D_t^S, h_t^R)$ for all config x_t $\forall\, t$
3. select the config/equilibrium of minimal cost $C_t \gamma_t(q_t, x_t)$ $\forall\, t$
4. stop if $h_{t+1}^R \approx h_t^R + q_t^R$ or update h^R

SEARCH THE CONTINUOUS h^R-SPACE: IN PRACTICE

Step 2: compute all equilibria $(q_t, h_t) \in Eq(x_t, D_t^S, h_t^R)$ for all config $x_t \; \forall t$

splitting the equilibrium problems in time and in space enables us to enumerate the sub-configurations



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Step 4: update tank level profiles h^R closer to satisfy both $h_{t+1}^R \approx h_t^R + q_t^R$ and $\underline{H}^R \leq h_t^R \leq \overline{H}^R$ $\forall t$

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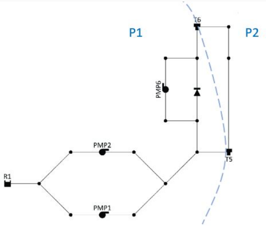
Step 0: compute initial tank level profiles h^R

we built a deep learning model to predict the optimal profiles from history

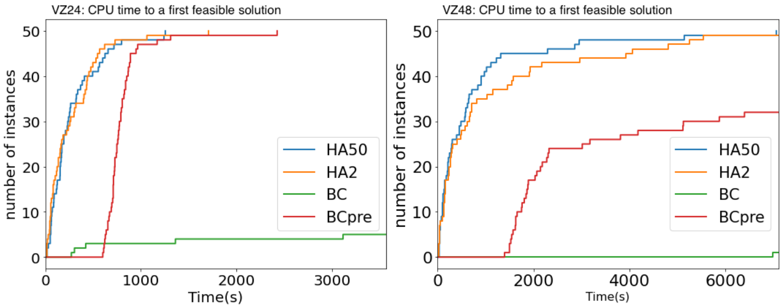
- using a (time) scaling mechanism to save on the training phase
- using Monte-Carlo dropouts to restart/diversify the search

Experimental evaluation



- 50 instances [VanZyl04]
- stop at the first feasible solution

- **HA**: deep learning + variable splitting
- **BC**: exact algorithm [Bonvin21] + **BCpre** preprocessing [Tavakoli22]



Conclusion

- integration of machine learning, simulation and optimization
- time and space decomposition
- reasoning on the implied storage state variables instead of the discrete decision control variables
- practical scalability ? theoretical convergence ?
- other applications in water management ?

References

- **S.Demassey, V. Sessa, A. Tavakoli** Deep learning and alternating direction method for discrete control with storage. In International Symposium on Combinatorial Optimization 2024.
- **A. Tavakoli, V. Sessa, S. Demassey** Strengthening mathematical models for pump scheduling in water distribution. In 14th International Conference on Applied Energy 2022.
- **G. Bonvin, S. Demassey, A. Lodi** Pump scheduling in drinking water distribution networks with an LP/NLP-based branch and bound. Optimization and Engineering 2021.
- **G. Bonvin, S. Demassey** Extended linear formulation of the pump scheduling problem in water distribution networks. In International Network Optimization Conference 2019.

- papers available at `https://sofdem.github.io/`
- code available at `https://github.com/sofdem/gopslpnlpbb`