Alternate Search for block-structured nonconvex MINLP

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MINLP FOR CLIMATE

Bingo !

decomposition methods

- combinatorics $2^{n/2} + 2^{n/2}$
- hybrid & recycle tools

(non)convex optimization + CO

- difference-of-convex $y - y^2 \leq 0$
- monotropic programming [Rockafellar'88]
- variable splitting and alternate projection
  e.g. Douglas-Rachford operator, ADMM, alternate convex search

Applications

load shifing for NL systems with storage

- unsync energy consumption/load service
- get more efficient operating points
- align consumption with energy surplus

ex: pump scheduling in water networks

with V. Sessa, A. Tavakoli, G. Bonvin, A. Lodi

traffic assignment and network design

- public infrastructures and traffic congestion

ex: discrete network design problem

with M. Levin, D. Rey

operating the power distribution grid

- stability when intermittent RES/new usages
- modulation/curtailment s.t. priority/fairness

ex: joint chance-constr discrete AC-OPF

with K. Syrtseva, P. Javal, W. de Oliveira

energy models in prospective analysis

- evaluate policies and guide political action
- long-term capacity expansion planning

ex: Markal-TIMES (IEA-ETSAP, 1980)

with G. Siggini, E. Assoumou, S. Selosse

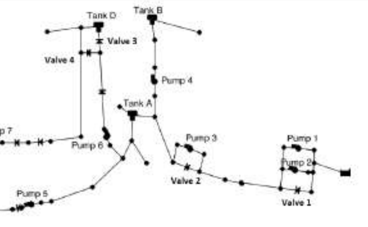
Common block structure

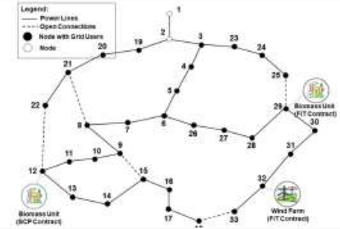
lower operation level: min nonlinear cost flows

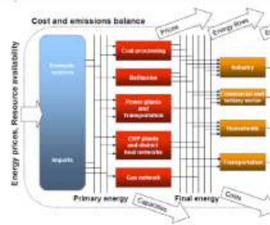
single (*water, power*) or multiple (*drivers, materials*) commodities

upper decision level: network configurations with coordination

- variable topology: arc interdiction *(switch on/off a pump, road/process investment)*
- variable boundary conditions *(uncertain demand, dynamic demand/supply)*







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interdependencies

- discrete time planning: sequence of stationary flows linked by investment decisions
- storage nodes: sequence-dependent sequence of flows
- traffic: flow-dependent individual travel times
- OPF: synchronized uncertainties and modulation

BILEVEL MODEL, MULTIPLE FOLLOWERS

$$\min_{y,x,u} \sum_k cost(y_k, x_k)$$

$$\begin{aligned} s.t.\ x_k &\in mincostflow(y_k, u_k) && \forall k \\ u_k &= state(u_{\neg k}, x_{\neg k}) && \forall k. \end{aligned}$$

- x arc flow
- y binary arc interdiction/activation
- u implied boundary conditions

ex: pump scheduling

$$\min_{y0/1,x,u} \sum_t c_t(y_t, x_t)$$

$$\begin{aligned} s.t. x_t &\in mincostflow(y_t, D_t, u_t) && \forall t \\ u_{t+1} &= u_t + \sigma^\top x_t && \forall t \\ \underline{U}_t \le u_t &\le \overline{U}_t && \forall t. \end{aligned}$$

- known demand D_t, dynamic tariff c_t at each time t
- flow x_t when switching pumps y_t given tank levels u_t
- sequence dependency $u_t \longrightarrow x_t \longrightarrow u_{t+1}$
- tank levels u_t with tight bounds $\Longrightarrow$ feasibility issue

mincost flow: MONOTROPIC PROGRAM

primal (distribution): x solves

$$\mathscr{P}: \min_x \sum_a f_a(x_a) + u_R^\top E_R^\top x$$

$$s.t.\ E_S^\top x = D_S$$

- with f_a l.s.c; in apps: f_a (energy dissipation) smooth, strictly convex $\Rightarrow$ unique flow x

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KKT (equilibrium): (x, u) solves

$$\mathscr{E}: E_S^\top x = D_S$$
$$v_a = u_i - u_j = f_a'(x_a) \qquad \forall\, a = (i,j)$$

dual (differential): u solves

$$\mathscr{D}: \min_u \sum_a f_a^*(v_a) + D_S^\top u_S$$
$$s.t.\ v := -Eu$$

strong duality: (x, u) solves

$$\mathscr{S}: E_S^\top x = D_S, v := -Eu$$
$$\sum_a \left(f_a(x_a) + f_a^*(v_a)\right) + u_R^\top E_R^\top x + D_S^\top u_S \leq 0.$$

- with f_a l.s.c; in apps: f_a (energy dissipation) smooth, strictly convex $\Rightarrow$ unique flow x
- f_a' resistance (potential loss), u_S potential (pressure, voltage, Wardrop's node price)
- $f_a^* = \int f_a'^{-1}$: $f_a^*(v_a) = -f_a(f_a'^{-1}(v_a)) + v_a f_a'^{-1}(v_a)$ convex conjugate: not polynomial

FEASIBLE SOLUTION FOR PUMP SCHEDULING

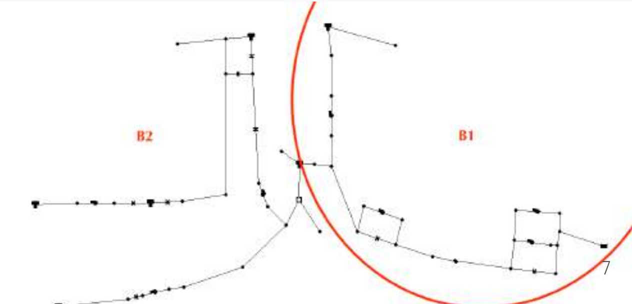
$$\min_{y0/1,x,u} \sum_t c_t(y_t, x_t) \quad (1)$$

$$s.t. x_t \in mincostflow(y_t, D_t, u_t) \quad \forall t \quad (2)$$

$$u_{t+1} = u_t + \sigma^\top x_t \quad \forall t \quad (3)$$

$$\underline{U}_t \leq u_t \leq \overline{U}_t \quad \forall t. \quad (4)$$

- solutions in discrete space y are sparse and scarce
- dualizing time coupling (3) in LR [Ghaddar'15] or variable copy $u = U$ in ADMM [Ulusoy'25] does not fix u_t
- dualizing (3) and fixing level variables u splits in both time and space, with few y_t variables in each component: enumerate and solve $mincostflow$ independently



SEARCH THE u-SPACE

$$\min_{u \in U} z(u) = \min_{y0/1,x} \sum_t c_t(y_t, x_t) + \mu_t(u_{t+1} - u_t - \sigma^\top x_t)$$

$$s.t. x_t \in mincostflow(y_t, D_t, u_t) \qquad \forall t.$$

- z: first-order information, smoothness, convexity ?
- alternate convex search:
  1/ $P(u^j)$: fix $u = u^j$ get (y^j, x^j)
  2/ $P(y^j, x^j)$: fix $(y, x) = (y^j, x^j)$ get u^{j+1}
  3/ (update μ)

partial split

- **keep** $mincostflow$ (2) in $P(u)$ as a constraint but **drop it** from $P(y, x)$
- start from a (learned) trial point u^0, **repair** feasibility by alternate search
- penalty/multipliers update policy: **ADMM** [Boyd'00] or **PADM** [Geißler'17]

OPTION 1: PARTIAL SPLIT AND PADM-LIKE

given penalty vector μ, increase μ when $||u^j - u^{j+1}||_\infty \leq \epsilon$,

1: fix levels u, then compute (y, x)

$$P(u) : \min_{(y,x)} \sum_t c_t(y_t, x_t) + \mu_t^\top ||u_{t+1} - u_t - \sigma^\top x_t||_1$$

$$s.t. : x_t \in mincostflow(y_t, D_t, u_t) \forall\, t.$$

solve $mincostflow_t$ independently on any graph component b, 0/1 vector y_{tb}

$\downarrow \quad \uparrow$ stop when $||u_{t+1} - u_t - \sigma^\top x_t||_\infty < \epsilon$

2: fix command (y, x), then compute u

$$P(y, x) : \min_u \sum_t c_t(x_t, y_t) + \mu_t^\top ||u_{t+1} - u_t - \sigma^\top x_t||_1 : u \in [\underline{U}, \overline{U}]$$

OPTION 2: PARTIAL SPLIT AND ADMM-LIKE

Given multipliers μ and penalty ρ:

1: fix levels u, then compute (y, x)

$$P(u) : \min_{(y,x)} \sum_t c_t(y_t, x_t) + \mu_t^\top (u_{t+1} - u_t - \sigma^\top x_t) + \frac{\rho}{2} ||u_{t+1} - u_t - \sigma^\top x_t||_2^2$$

$$s.t. : x_t \in mincostflow(y_t, D_t, u_t) \forall t \forall b.$$

ℓ_2-regularization is separable here

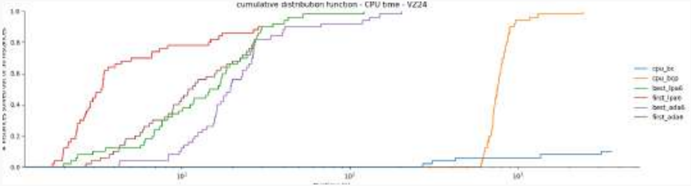
2: fix command (y, x), then compute u

$$P(y, x) : \min_{u \in U} \sum_t c_t(x_t, y_t) + \mu_t^\top (u_{t+1} - u_t - \sigma^\top x_t) + \frac{\rho}{2} ||u_{t+1} - u_t - \sigma^\top x_{tb}||_2^2$$

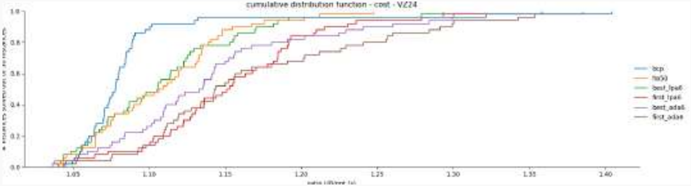
3: update $\mu_t = \mu_t + \rho * (u_{t+1} - u_t - \sigma^\top x_t)$

EXPERIMENTS: LEARNED PROFILES u + PARTIAL SPLIT

- 30 initial trials u (deep learning + MonteCarlo dropout) + interpolation $T = 12 \rightarrow 24$
- stop at first feasible solution, or best within 30s [D., Sessa, Tavakoli'24] (no parallelization)
- compare with first solution from SOA Branch-and-Check [Bonvin, D., Lodi'21] w/wo advanced preprocessing [Tavakoli, D., Sessa'22] on Van Zyl benchmark



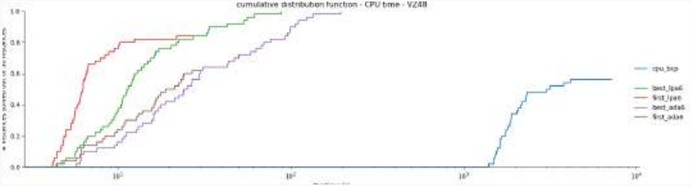
T=24 cpu log(s)



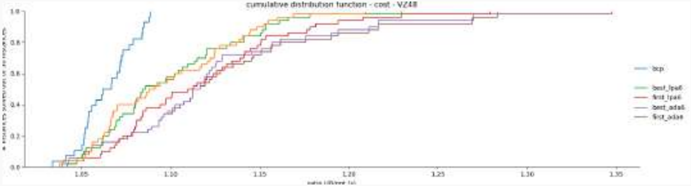
T=24 opt gap

EXPERIMENTS: LEARNED PROFILES u + PARTIAL SPLIT

- 30 initial trials u (deep learning + MonteCarlo dropout) + interpolation $T = 12 \rightarrow 48$
- stop at first feasible solution, or best within 30s [D., Sessa, Tavakoli'24] (no parallelization)
- compare with first solution from SOA Branch-and-Check [Bonvin, D., Lodi'21] w/wo advanced preprocessing [Tavakoli, D., Sessa'22] on Van Zyl benchmark



T=48 cpu log(s)



T=48 opt gap

OPTION 3: FULL SPLIT ON THE STRONG-DUALITY MODEL

step 1: fix level u_R, then compute schedule and flow (y, x)

$$w(u_R): \min_{(y,x)} \sum_t c_t(y_t, x_t) + \mu_t^\top (u_{t+1} - u_t - \sigma^\top x_{tR}) + \lambda_t SD_t(x_t, u_t): \ (1 - y_t)x_t = 0,\ x_{tS} = D_{tS} \forall t$$

with $SD_t(x_t, u_t) = \sum_a f_a(x_{ta}) + f_a^*(v_{ta}) + u_{tR}^\top x_{tR} + D_{tS}^\top u_{tS}$ and $v_t = -Eu_t$

$w(u_R)$ remains separable in time and space

each is **separable in primal (x) /dual (u_S) parts**, corresponding each to a follower $mincost\,flow$ augmented with leader costs c and multipliers μ, λ:

perturbed primal

$$\mathscr{P}_t(y_t, u_{tR}): \min_{x_t} \lambda_t f(x_t) + (\lambda_t u_{tR} - \mu_t + c_t^1)^\top x_t$$

$$s.t.: x_{tS} = D_{tS},\ (1 - y_t)^\top x_t = 0.$$

perturbed dual

$$\mathscr{D}_t(y_t, u_{tR}): \min_{u_{tS}} \lambda_t f^*(v_t) + \lambda_t D_t^\top u_{tS}$$

$$s.t.: v_t = -Eu_t.$$

CONCLUSION

- fixing coupling variables vs relaxing coupling constraints: keep structure, split deeper (time/space/primal-dual), linearize bilinear terms
- alternative bilevel view: (leader) implied continuous storage variables (follower) discrete decisions
- alternative ML/MIP hybrid: ML for optimality, MIP for feasibility
- generalization to bilevel programming and MPEC (with Antonio Sasaki and Valentina Sessa)
- convergence for partial split ?

References

- **Rockafellar** on nonlinear flows, **Eckstein**, **Rockafellar** on monotone operators
- **Boyd** on proximal algorithms, apps of ADMM in ML, image processing, and for OPF
- biconvex optimization [Gorski,Pfeuffer,Klamroth'07]

Combinatorial Optimization:

- The feasibility pump [Fischetti,Glover,Lodi'05], PADM for MIP [Geißler,Morsi,Schewe,Schmidt'17]
- Application to gas transportation problems [Geißler,Morsi,Schewe,Schmidt'15 and '18]
- Computing feasible points of bilevel problems with PADM [Kleinert,Schmidt'20]
- Application to pump scheduling [D.,Sessa,Tavakoli'24], [Ulusoy,Stoianov'25]

- **S.D., V. Sessa, A. Tavakoli** DL and alternating direction method for discrete control with storage. ISCO 2024.
- **A. Tavakoli, V. Sessa, S. D.** Strengthening mathematical models for pump scheduling. ICAE 2022.
- **G. Bonvin, S. D., A. Lodi** Pump scheduling in water networks with an LP/NLP-based B&B. Opt&Eng 2021.
- **G. Bonvin, S. D.** Extended linear formulation of the pump scheduling problem in water networks. INOC 2019.
- papers available at `https://sofdem.github.io/` code at `https://github.com/sofdem/`