5.62 change_continuity

DESCRIPTION LINKS GRAPH AUTOMATON

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Constraint

change_continuity(NB_PERIOD_CHANGE, NB_PERIOD_CONTINUITY, MIN_SIZE_CHANGE, MAX_SIZE_CHANGE, MIN_SIZE_CONTINUITY, MAX_SIZE_CONTINUITY, NB_CHANGE, NB_CONTINUITY, VARIABLES, CTR)

Arguments

NB_PERIOD_CHANGE	:	dvar
NB_PERIOD_CONTINUITY	:	dvar
MIN_SIZE_CHANGE	:	dvar
MAX_SIZE_CHANGE	:	dvar
MIN_SIZE_CONTINUITY	:	dvar
MAX_SIZE_CONTINUITY	:	dvar
NB_CHANGE	:	dvar
NB_CONTINUITY	:	dvar
VARIABLES	:	collection(var−dvar)
CTR	:	atom

Restrictions

NB_PERIOD_CHANGE ≥ 0
NB_PERIOD_CONTINUITY ≥ 0
MIN_SIZE_CHANGE ≥ 0
MAX_SIZE_CHANGE $\geq$ MIN_SIZE_CHANGE
MIN_SIZE_CONTINUITY ≥ 0
MAX_SIZE_CONTINUITY $\geq$ MIN_SIZE_CONTINUITY
NB_CHANGE ≥ 0
NB_CONTINUITY ≥ 0
required(VARIABLES, var)
CTR $\in [=, \neq, <, \geq, >, \leq]$

On the one hand a *change* is defined by the fact that constraint $\texttt{VARIABLES}[i].\texttt{var}$ CTR $\texttt{VARIABLES}[i+1].\texttt{var}$ holds.
On the other hand a *continuity* is defined by the fact that constraint $\texttt{VARIABLES}[i].\texttt{var}$ CTR $\texttt{VARIABLES}[i+1].\texttt{var}$ does not hold.
A *period of change* on variables

$$\texttt{VARIABLES}[i].\texttt{var}, \texttt{VARIABLES}[i+1].\texttt{var}, \ldots, \texttt{VARIABLES}[j].\texttt{var}\ (i<j)$$

is defined by the fact that all constraints $\texttt{VARIABLES}[k].\texttt{var}$ CTR $\texttt{VARIABLES}[k+1].\texttt{var}$ hold for $k \in [i, j-1]$.
A *period of continuity* on variables

$$\texttt{VARIABLES}[i].\texttt{var}, \texttt{VARIABLES}[i+1].\texttt{var}, \ldots, \texttt{VARIABLES}[j].\texttt{var}\ (i<j)$$

is defined by the fact that all constraints $\texttt{VARIABLES}[k].\texttt{var}$ CTR $\texttt{VARIABLES}[k+1].\texttt{var}$ do not hold for $k \in [i, j-1]$.

Purpose

The constraint `change_continuity` holds if and only if:

- NB_PERIOD_CHANGE is equal to the number of periods of change,
- NB_PERIOD_CONTINUITY is equal to the number of periods of continuity,
- MIN_SIZE_CHANGE is equal to the number of variables of the smallest period of change,
- MAX_SIZE_CHANGE is equal to the number of variables of the largest period of change,
- MIN_SIZE_CONTINUITY is equal to the number of variables of the smallest period of continuity,
- MAX_SIZE_CONTINUITY is equal to the number of variables of the largest period of continuity,
- NB_CHANGE is equal to the total number of changes,
- NB_CONTINUITY is equal to the total number of continuities.

Example

$$(3, 2, 2, 4, 2, 4, 6, 4, \langle 1, 3, 1, 8, 8, 4, 7, 7, 7, 7, 2\rangle, \neq)$$

Figure 5.141 makes clear the different parameters that are associated with the given example for the collection $\texttt{VARIABLES} = \langle 1, 3, 1, 8, 8, 4, 7, 7, 7, 7, 2\rangle$. We place character — for representing a change and a blank for a continuity. On top of the solution we represent the different periods of change, while below we show the different periods of continuity. The `change_continuity` constraint holds since:

- Its number of periods of change NB_PERIOD_CHANGE is equal to 3 (i.e., the 3 periods depicted on top of Figure 5.141),
- Its number of periods of continuity NB_PERIOD_CONTINUITY is equal to 2 (i.e., the 2 periods depicted below Figure 5.141),
- The number of variables of its smallest period of change MIN_SIZE_CHANGE is equal to 2 (i.e., the number of variables involved in the third period of change 7 2 depicted on top of Figure 5.141),

- The number of variables of the largest period of change MAX_SIZE_CHANGE is equal to 4 (i.e., the number of variables involved in the first period of change $1\ 3\ 1\ 8$ depicted on top of Figure 5.141),
- The number of variables of the smallest period of continuity MIN_SIZE_CONTINUITY is equal to 2 (i.e., the number of variables involved in the first period $8\ 8$ depicted below Figure 5.141),
- The number of variables of the largest period of continuity MAX_SIZE_CONTINUITY is equal to 4 (i.e., the number of variables involved in the second period $7\ 7\ 7\ 7$ depicted below Figure 5.141),
- The total number of changes NB_CHANGE is equal to 6 (i.e., the number of occurrences of character — in Figure 5.141),
- The total number of continuities NB_CONTINUITY is equal to 4.

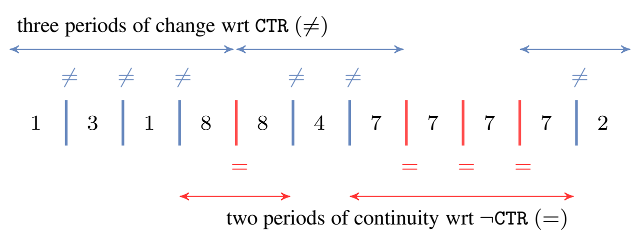


Figure 5.141: Illustration of the **Example** slot: periods of changes and periods of continuities wrt the constraint CTR equal to $\neq$

Typical

NB_PERIOD_CHANGE > 0
NB_PERIOD_CONTINUITY > 0
MIN_SIZE_CHANGE > 0
MIN_SIZE_CONTINUITY > 0
NB_CHANGE > 0
NB_CONTINUITY > 0
$|$VARIABLES$| > 1$
range(VARIABLES.var) > 1
CTR $\in [\neq]$

Symmetry

One and the same constant can be added to the var attribute of all items of VARIABLES.

Arg. properties

- Functional dependency: NB_PERIOD_CHANGE determined by VARIABLES and CTR.
- Functional dependency: NB_PERIOD_CONTINUITY determined by VARIABLES and CTR.
- Functional dependency: MIN_SIZE_CHANGE determined by VARIABLES and CTR.
- Functional dependency: MAX_SIZE_CHANGE determined by VARIABLES and CTR.
- Functional dependency: MIN_SIZE_CONTINUITY determined by VARIABLES and CTR.
- Functional dependency: MAX_SIZE_CONTINUITY determined by VARIABLES and CTR.
- Functional dependency: NB_CHANGE determined by VARIABLES and CTR.
- Functional dependency: NB_CONTINUITY determined by VARIABLES and CTR.

Remark

If the variables of the collection VARIABLES have to take distinct values between 1 and the total number of variables, we have what is called a permutation. In this case, if we choose the binary constraint $<$, then MAX_SIZE_CHANGE gives the size of the longest run of the permutation; A *run* is a maximal increasing contiguous subsequence in a permutation.

See also

common keyword: group, group_skip_isolated_item, stretch_path *(timetabling constraint)*.

Keywords

characteristic of a constraint: automaton, automaton with counters, automaton with same input symbol.

combinatorial object: sequence, run of a permutation, permutation.

constraint arguments: reverse of a constraint.

constraint network structure: sliding cyclic(1) constraint network(2), sliding cyclic(1) constraint network(3).

constraint type: timetabling constraint.

filtering: glue matrix.

final graph structure: connected component, apartition, acyclic, bipartite, no loop.

modelling: functional dependency.

Arc input(s)	VARIABLES
Arc generator	$PATH \mapsto$ collection(variables1, variables2)
Arc arity	2
Arc constraint(s)	variables1.var CTR variables2.var
Graph property(ies)	• **NCC**= NB_PERIOD_CHANGE • **MIN_NCC**= MIN_SIZE_CHANGE • **MAX_NCC**= MAX_SIZE_CHANGE • **NARC**= NB_CHANGE
Graph class	• ACYCLIC • BIPARTITE • NO_LOOP

Arc input(s)	VARIABLES
Arc generator	$PATH \mapsto$ collection(variables1, variables2)
Arc arity	2
Arc constraint(s)	variables1.var¬ CTR variables2.var
Graph property(ies)	• **NCC**= NB_PERIOD_CONTINUITY • **MIN_NCC**= MIN_SIZE_CONTINUITY • **MAX_NCC**= MAX_SIZE_CONTINUITY • **NARC**= NB_CONTINUITY
Graph class	• ACYCLIC • BIPARTITE • NO_LOOP

Graph model

We use two graph constraints to respectively catch the constraints on the period of changes and of the period of continuities. In both case each period corresponds to a connected component of the final graph.

Parts (A) and (B) of Figure 5.142 respectively show the initial and final graph associated with the first graph constraint of the **Example** slot.

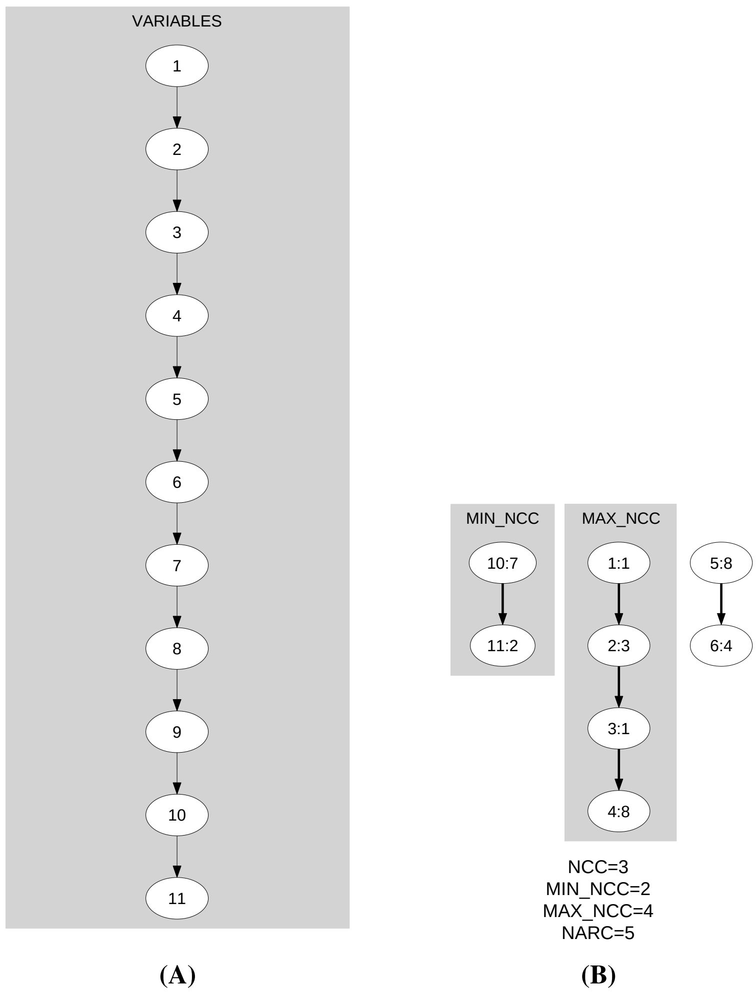
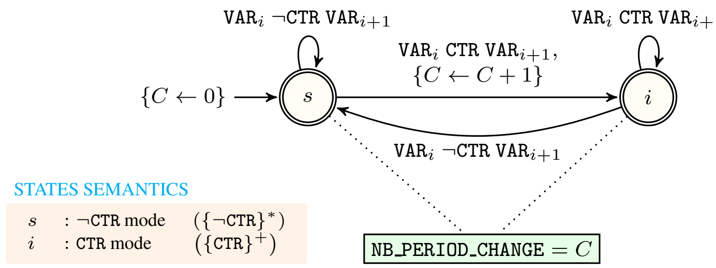


Figure 5.142: Initial and final graph of the `change_continuity` constraint

Automaton

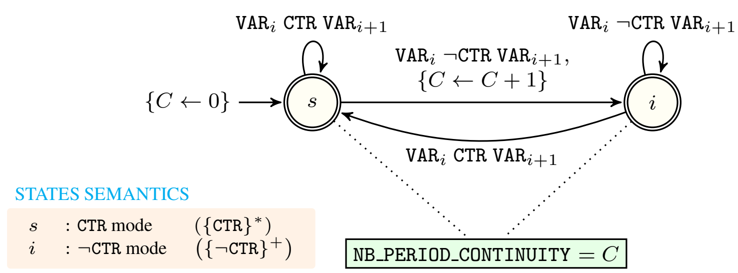
Figures 5.143 , 5.144 , 5.147 , 5.148 , 5.151 , 5.152 and 5.155 depict the automata associated with the different graph parameters of the `change_continuity` constraint. For the automata that respectively compute NB_PERIOD_CHANGE, NB_PERIOD_CONTINUITY MIN_SIZE_CHANGE, MIN_SIZE_CONTINUITY MAX_SIZE_CHANGE, MAX_SIZE_CONTINUITY NB_CHANGE and NB_CONTINUITY we have a 0-1 signature variable S_i for each pair of consecutive variables $(\mathtt{VAR}_i, \mathtt{VAR}_{i+1})$ of the collection VARIABLES. The following signature constraint links $\mathtt{VAR}_i$, $\mathtt{VAR}_{i+1}$ and S_i: $\mathtt{VAR}_i$ CTR $\mathtt{VAR}_{i+1} \Leftrightarrow S_i$.



Glue matrix where $\overrightarrow{C}$ and $\overleftarrow{C}$ resp. represent the counters values C at the end of a prefix and at the end of the corresponding reverse suffix that partitions the sequence VARIABLES.

	s	i
s	$\overrightarrow{C}+\overleftarrow{C}$	$\overrightarrow{C}+\overleftarrow{C}$
i	$\overrightarrow{C}+\overleftarrow{C}$	$\overrightarrow{C}+\overleftarrow{C}-1$

Figure 5.143: Automaton for the NB_PERIOD_CHANGE argument of the `change_continuity` constraint and its glue matrix; note that the reverse of `change_continuity` with CTR $\in \{=, \neq\}$ is the same constraint, while the reverse with CTR $\in \{<\}$ (resp. CTR $\in \{\leq\}$) is CTR $\in \{>\}$ (resp. CTR $\in \{\geq\}$).



Glue matrix where $\overrightarrow{C}$ and $\overleftarrow{C}$ resp. represent the counters values C at the end of a prefix and at the end of the corresponding reverse suffix that partitions the sequence VARIABLES.

	s	i
s	$\overrightarrow{C}+\overleftarrow{C}$	$\overrightarrow{C}+\overleftarrow{C}$
i	$\overrightarrow{C}+\overleftarrow{C}$	$\overrightarrow{C}+\overleftarrow{C}-1$

Figure 5.144: Automaton for the NB_PERIOD_CONTINUITY argument of the `change_continuity` constraint and its glue matrix; note that the reverse of `change_continuity` with CTR $\in \{=, \neq\}$ is the same constraint, while the reverse with CTR $\in \{<\}$ (resp. CTR $\in \{\leq\}$) is CTR $\in \{>\}$ (resp. CTR $\in \{\geq\}$).

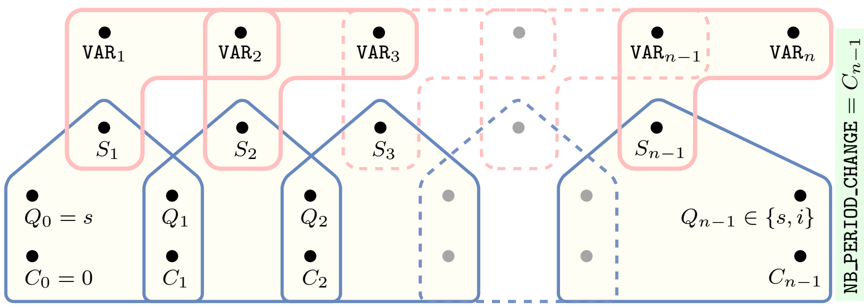


Figure 5.145: Hypergraph of the reformulation corresponding to the automaton of the NB_PERIOD_CHANGE argument of the change_continuity constraint

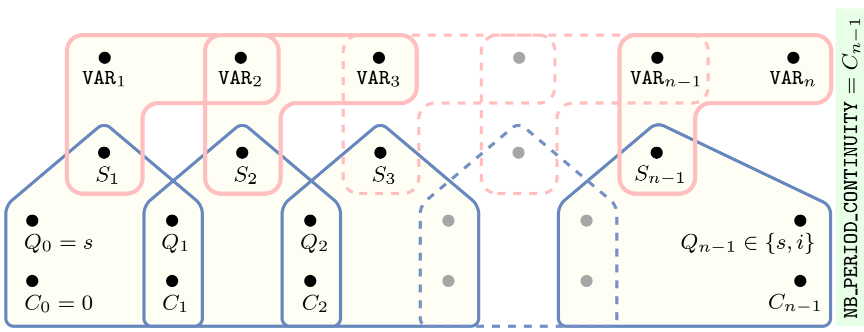
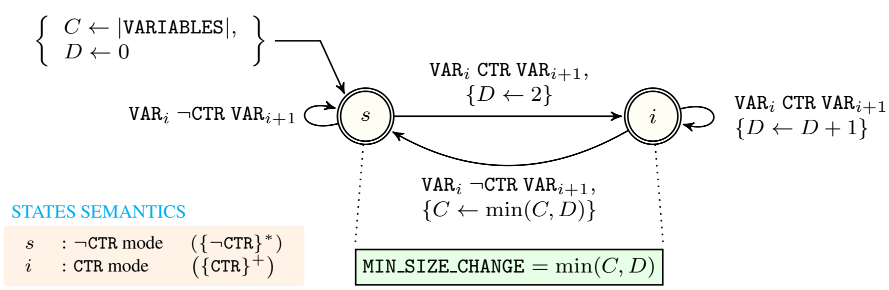


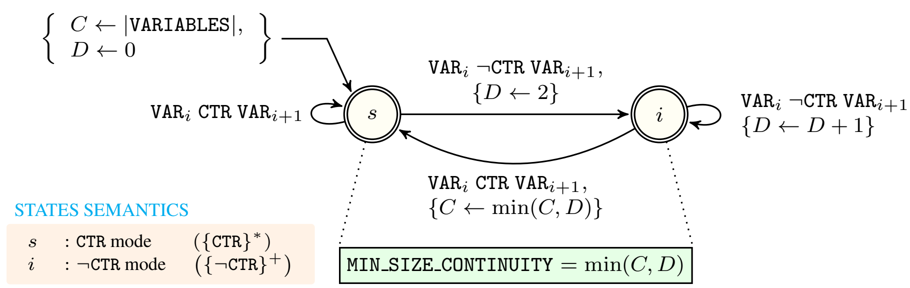
Figure 5.146: Hypergraph of the reformulation corresponding to the automaton of the NB_PERIOD_CONTINUITY argument of the change_continuity constraint



Glue matrix where $\overrightarrow{C}$, $\overrightarrow{D}$ and $\overleftarrow{C}$, $\overleftarrow{D}$ resp. represent the counters values C, D at the end of a prefix and at the end of the corresponding reverse suffix that partitions the sequence VARIABLES.

	s	i
s	$\min(\overrightarrow{C}, \overrightarrow{D} + \overleftarrow{D}, \overleftarrow{C})$	$\min(\overrightarrow{C}, \overleftarrow{D}, \overleftarrow{C})$
i	$\min(\overrightarrow{C}, \overrightarrow{D}, \overleftarrow{C})$	$\min(\overrightarrow{C}, \overrightarrow{D} + \overleftarrow{D} - 1, \overleftarrow{C})$

Figure 5.147: Automaton for the MIN_SIZE_CHANGE argument of the change_continuity constraint; its glue matrix when CTR $\in \{=, \neq\}$.



Glue matrix where $\overrightarrow{C}$, $\overrightarrow{D}$ and $\overleftarrow{C}$, $\overleftarrow{D}$ resp. represent the counters values C, D at the end of a prefix and at the end of the corresponding reverse suffix that partitions the sequence VARIABLES.

	s	i
s	$\min(\overrightarrow{C}, \overrightarrow{D}+\overleftarrow{D}, \overleftarrow{C})$	$\min(\overrightarrow{C}, \overleftarrow{D}, \overleftarrow{C})$
i	$\min(\overrightarrow{C}, \overrightarrow{D}, \overleftarrow{C})$	$\min(\overrightarrow{C}, \overrightarrow{D}+\overleftarrow{D}-1, \overleftarrow{C})$

Figure 5.148: Automaton for the MIN_SIZE_CONTINUITY argument of the change_continuity constraint; its glue matrix when CTR $\in \{=, \neq\}$.

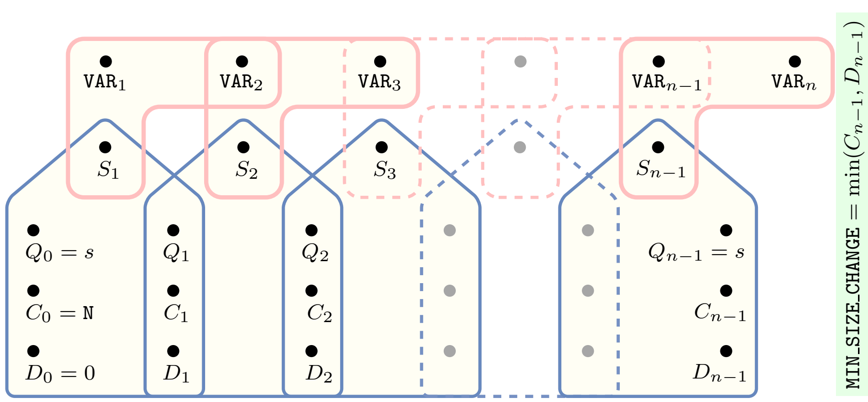


Figure 5.149: Hypergraph of the reformulation corresponding to the automaton of the MIN_SIZE_CHANGE argument of the change_continuity constraint where N stands for |VARIABLES| (since all states of the automaton are accepting there is no restriction on the last variable Q_{n-1})

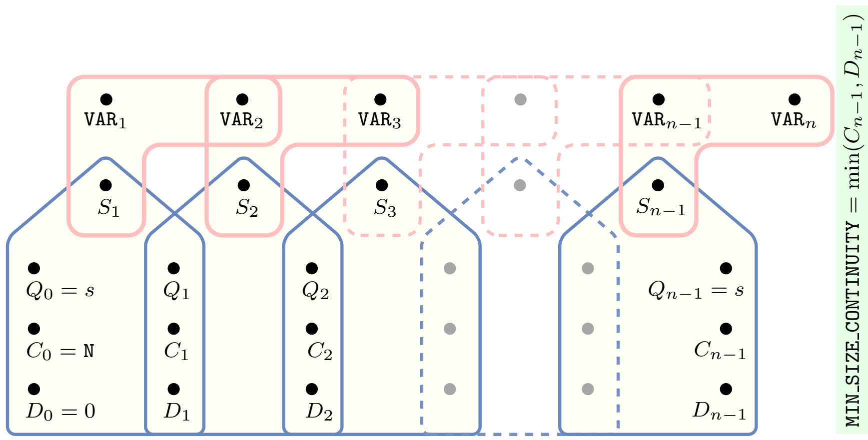
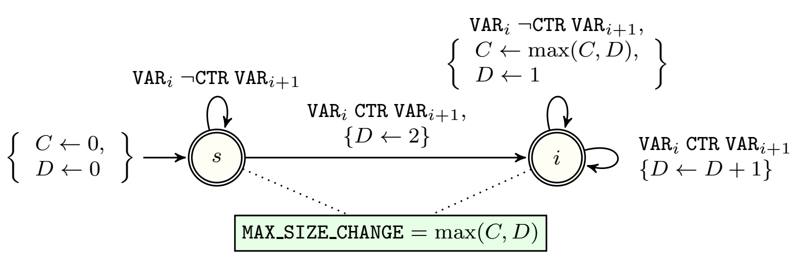


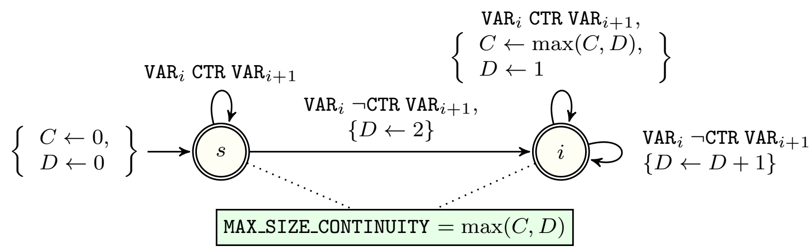
Figure 5.150: Hypergraph of the reformulation corresponding to the automaton of the `MIN_SIZE_CONTINUITY` argument of the `change_continuity` constraint where `N` stands for $|$`VARIABLES`$|$ (since all states of the automaton are accepting there is no restriction on the last variable Q_{n-1})



Glue matrix where $\overrightarrow{C}$, $\overrightarrow{D}$ and $\overleftarrow{C}$, $\overleftarrow{D}$ resp. represent the counters values C, D at the end of a prefix and at the end of the corresponding reverse suffix that partitions the sequence `VARIABLES`.

	s	i
s	0	$\max\left(\overleftarrow{D}, \overleftarrow{C}\right)$
i	$\max\left(\overrightarrow{C}, \overrightarrow{D}\right)$	$\max\left(\overrightarrow{C}, \overrightarrow{D} + \overleftarrow{D} - 1, \overleftarrow{C}\right)$

Figure 5.151: Automaton for the `MAX_SIZE_CHANGE` argument of the `change_continuity` constraint; its glue matrix when `CTR` $\in \{=, \neq\}$.



Glue matrix where $\overrightarrow{C}$, $\overrightarrow{D}$ and $\overleftarrow{C}$, $\overleftarrow{D}$ resp. represent the counters values C, D at the end of a prefix and at the end of the corresponding reverse suffix that partitions the sequence VARIABLES.

	s	i
s	0	$\max\left(\overleftarrow{D}, \overleftarrow{C}\right)$
i	$\max\left(\overrightarrow{C}, \overrightarrow{D}\right)$	$\max\left(\overrightarrow{C}, \overrightarrow{D} + \overleftarrow{D} - 1, \overleftarrow{C}\right)$

Figure 5.152: Automaton for the MAX_SIZE_CONTINUITY argument of the change_continuity constraint; its glue matrix when CTR $\in \{=, \neq\}$.

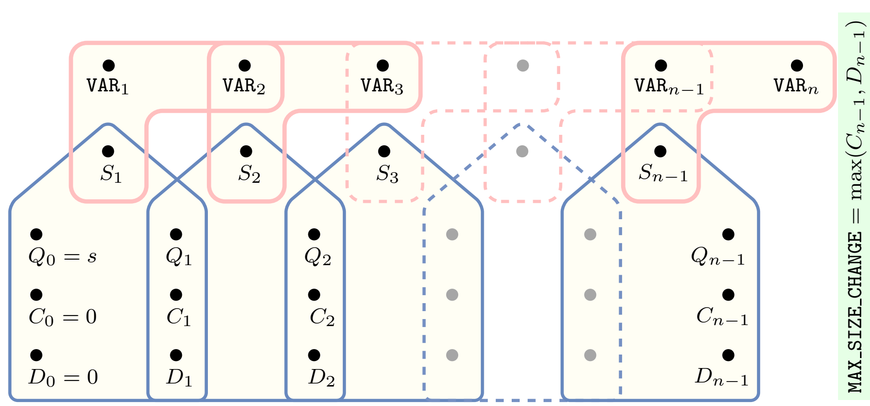


Figure 5.153: Hypergraph of the reformulation corresponding to the automaton of the MAX_SIZE_CHANGE argument of the change_continuity constraint (since all states of the automaton are accepting there is no restriction on the last variable Q_{n-1})

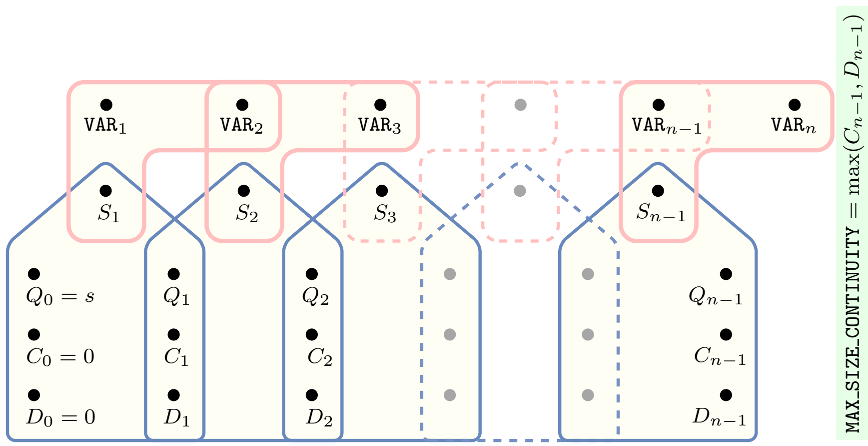
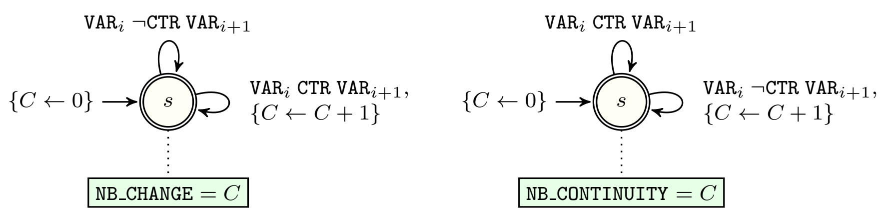


Figure 5.154: Hypergraph of the reformulation corresponding to the automaton of the MAX_SIZE_CONTINUITY argument of the change_continuity constraint (since all states of the automaton are accepting there is no restriction on the last variable Q_{n-1})



Common glue matrix where $\overrightarrow{C}$ and $\overleftarrow{C}$ resp. represent the counters values C at the end of a prefix and at the end of the corresponding reverse suffix that partitions the sequence VARIABLES.

	s
s	$\overrightarrow{C} + \overleftarrow{C}$

Figure 5.155: Automata for the NB_CHANGE and NB_CONTINUITY arguments of the change_continuity constraint; their common glue matrix when $\arg CTR \in \{=, \neq\}$.

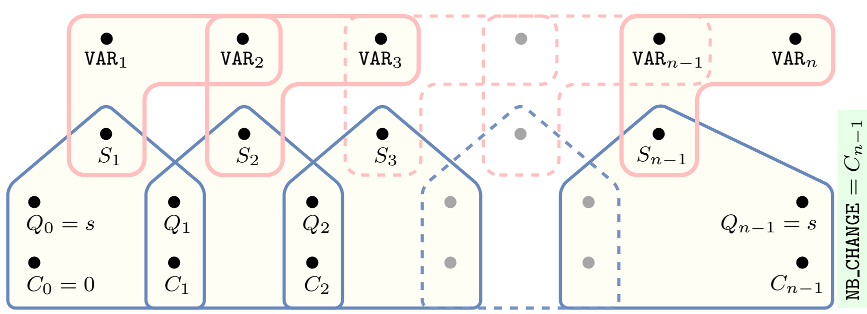


Figure 5.156: Hypergraph of the reformulation corresponding to the automaton of the NB_CHANGE argument of the change_continuity constraint

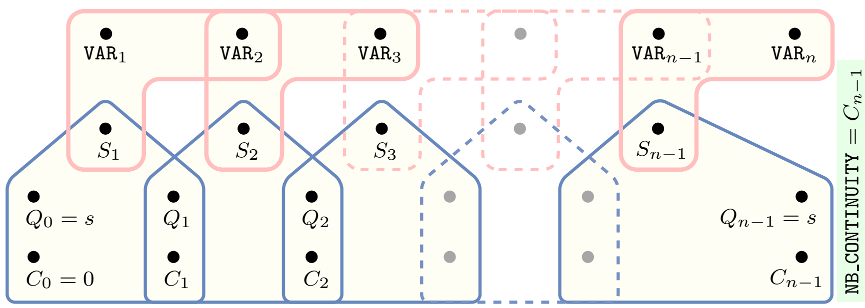


Figure 5.157: Hypergraph of the reformulation corresponding to the automaton of the NB_CONTINUITY argument of the change_continuity constraint