5.93   coveredby_sboxes

Origin   Geometry, derived from [338]

Constraint   coveredby_sboxes(K, DIMS, OBJECTS, SBOXES)

Synonym   coveredby.

Types   VARIABLES : collection(v−dvar)
                    INTEGERS   : collection(v−int)
                    POSITIVES : collection(v−int)

Arguments   K   : int
                    DIMS   : sint
                    OBJECTS : collection(oid−int, sid−dvar, x − VARIABLES)
                    SBOXES   : collection(sid−int, t − INTEGERS, l − POSITIVES)

Restrictions $|\text{VARIABLES}| \geq 1$
                    $|\text{INTEGERS}| \geq 1$
                    $|\text{POSITIVES}| \geq 1$
                    required(VARIABLES, v)
                    $|\text{VARIABLES}| = K$
                    required(INTEGERS, v)
                    $|\text{INTEGERS}| = K$
                    required(POSITIVES, v)
                    $|\text{POSITIVES}| = K$
                    $\text{POSITIVES.v} > 0$
                    $K > 0$
                    $\text{DIMS} \geq 0$
                    $\text{DIMS} < K$
                    increasing_seq(OBJECTS, [oid])
                    required(OBJECTS, [oid, sid, x])
                    $\text{OBJECTS.oid} \geq 1$
                    $\text{OBJECTS.oid} \leq |\text{OBJECTS}|$
                    $\text{OBJECTS.sid} \geq 1$
                    $\text{OBJECTS.sid} \leq |\text{SBOXES}|$
                    required(SBOXES, [sid, t, l])
                    $|\text{SBOXES}| \geq 1$
                    $\text{SBOXES.sid} \geq 1$
                    $\text{SBOXES.sid} \leq |\text{SBOXES}|$
                    do_not_overlap(SBOXES)

Purpose

Holds if, for each pair of objects (O_i, O_j), $i < j$, O_i is covered by O_j with respect to a set of dimensions depicted by DIMS. O_i and O_j are objects that take a shape among a set of shapes. Each *shape* is defined as a finite set of shifted boxes, where each shifted box is described by a box in a K-dimensional space at a given offset (from the origin of the shape) with given sizes. More precisely, a *shifted box* is an entity defined by its shape id sid, shift offset t, and sizes l. Then, a shape is defined as the union of shifted boxes sharing the same shape id. An *object* is an entity defined by its unique object identifier oid, shape id sid and origin x.

An object O_i *is covered by* an object O_j with respect to a set of dimensions depicted by DIMS if and only if, for all shifted box s_i of O_i, there exists a shifted box s_j of O_j such that:

- For all dimensions $d \in$ DIMS, (1) the start of s_j in dimension d is less than or equal to the start of s_i in dimension d, and (2) the end of s_i in dimension d is less than or equal to the end of s_j in dimension d.

- There exists a dimension d where, (1) the start of s_j in dimension d coincide with the start of s_i in dimension d, or (2) the end of s_j in dimension d coincide with the end of s_i in dimension d.

Example

$$
\left(
\begin{array}{l}
2, \{0, 1\}, \\
\left\langle
\begin{array}{lll}
\texttt{oid} - 1 & \texttt{sid} - 4 & \texttt{x} - \langle 2, 3 \rangle, \\
\texttt{oid} - 2 & \texttt{sid} - 2 & \texttt{x} - \langle 2, 2 \rangle, \\
\texttt{oid} - 3 & \texttt{sid} - 1 & \texttt{x} - \langle 1, 1 \rangle
\end{array}
\right\rangle, \\
\left\langle
\begin{array}{lll}
\texttt{sid} - 1 & \texttt{t} - \langle 0, 0 \rangle & \texttt{l} - \langle 3, 3 \rangle, \\
\texttt{sid} - 1 & \texttt{t} - \langle 3, 0 \rangle & \texttt{l} - \langle 2, 2 \rangle, \\
\texttt{sid} - 2 & \texttt{t} - \langle 0, 0 \rangle & \texttt{l} - \langle 2, 2 \rangle, \\
\texttt{sid} - 2 & \texttt{t} - \langle 2, 0 \rangle & \texttt{l} - \langle 1, 1 \rangle, \\
\texttt{sid} - 3 & \texttt{t} - \langle 0, 0 \rangle & \texttt{l} - \langle 2, 2 \rangle, \\
\texttt{sid} - 3 & \texttt{t} - \langle 2, 1 \rangle & \texttt{l} - \langle 1, 1 \rangle, \\
\texttt{sid} - 4 & \texttt{t} - \langle 0, 0 \rangle & \texttt{l} - \langle 1, 1 \rangle
\end{array}
\right\rangle
\end{array}
\right)
$$

Figure 5.209 shows the objects of the example. Since O_1 is covered by both O_2 and O_3, and since O_2 is covered by O_3, the coveredby_sboxes constraint holds.

Typical

$|\texttt{OBJECTS}| > 1$

Symmetries

- Items of SBOXES are permutable.

- Items of OBJECTS.x, SBOXES.t and SBOXES.l are permutable (*same permutation used*).

Remark

One of the eight relations of the *Region Connection Calculus* [338]. The constraint coveredby_sboxes is a restriction of the original relation since it requires that each shifted box of an object is covered by one shifted box of the other object.

See also

common keyword: contains_sboxes, covers_sboxes, disjoint_sboxes, equal_sboxes, inside_sboxes, meet_sboxes *(rcc8)*, non_overlap_sboxes *(geometrical constraint, logic)*, overlap_sboxes *(rcc8)*.

(A) Shape of the
third object

(B) Shapes of the
second object

(C) Shape of the
first object

OBJECTS

O_1:   oid − 1   sid − 4   x − $\langle 2,3 \rangle$
O_2:   oid − 2   sid − 2   x − $\langle 2,2 \rangle$
O_3:   oid − 3   sid − 1   x − $\langle 1,1 \rangle$

(D) Three objects O_1, O_2, O_3, where O_1 is covered by both O_2 and O_3
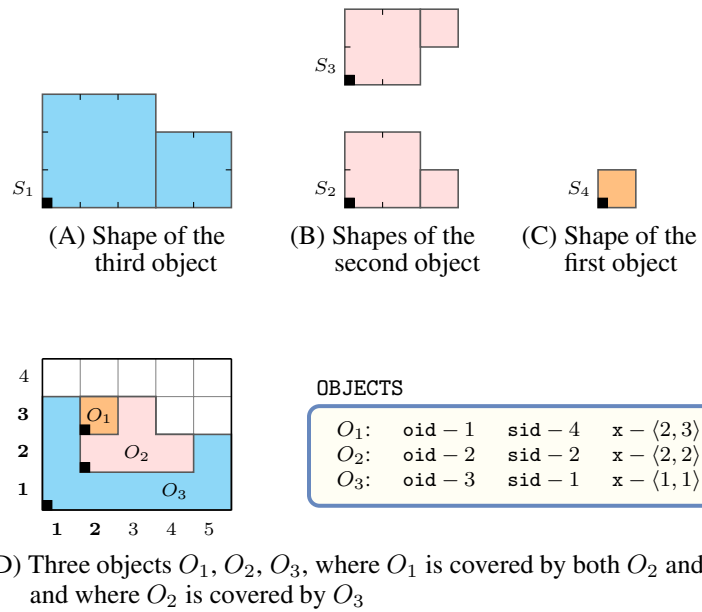and where O_2 is covered by O_3

Figure 5.209: (D) the three objects O_1, O_2, O_3 of the **Example** slot respectively as-
signed shapes S_4, S_2, S_1; (A), (B), (C) shapes S_1, S_2, S_3 and S_4 are respectively made
up from 2, 2, 2 and 1 single shifted box.

Logic

- $\mathtt{origin(O1, S1, D)} \stackrel{\mathrm{def}}{=} \mathtt{O1.x(D) + S1.t(D)}$
- $\mathtt{end(O1, S1, D)} \stackrel{\mathrm{def}}{=} \mathtt{O1.x(D) + S1.t(D) + S1.l(D)}$
- $\mathtt{coveredby\_sboxes(Dims, O1, S1, O2, S2)} \stackrel{\mathrm{def}}{=}$

$$\bigwedge \left(\begin{array}{l} \forall \mathtt{D} \in \mathtt{Dims} \\ \bigwedge \left(\begin{array}{l} \mathtt{origin} \left(\begin{array}{l} \mathtt{O2,} \\ \mathtt{S2,} \\ \mathtt{D} \end{array} \right) \leq \\ \mathtt{origin} \left(\begin{array}{l} \mathtt{O1,} \\ \mathtt{S1,} \\ \mathtt{D} \end{array} \right) \\ \mathtt{end(O1, S1, D)} \leq \\ \mathtt{end(O2, S2, D)} \end{array} \right), \\ \exists \mathtt{D} \in \mathtt{Dims} \\ \bigvee \left(\begin{array}{l} \mathtt{origin} \left(\begin{array}{l} \mathtt{O2,} \\ \mathtt{S2,} \\ \mathtt{D} \end{array} \right) = \\ \mathtt{origin} \left(\begin{array}{l} \mathtt{O1,} \\ \mathtt{S1,} \\ \mathtt{D} \end{array} \right) \\ \mathtt{end(O1, S1, D)} = \\ \mathtt{end(O2, S2, D)} \end{array} \right), \end{array} \right)$$

- $\mathtt{coveredby\_objects(Dims, O1, O2)} \stackrel{\mathrm{def}}{=}$
  $\forall \mathtt{S1} \in \mathtt{sboxes([O1.sid])}$
    $\exists \mathtt{S2} \in \mathtt{sboxes} \left(\left[\mathtt{O2.sid} \right] \right)$
      $\mathtt{coveredby\_sboxes} \left(\begin{array}{l} \mathtt{Dims,} \\ \mathtt{O1,} \\ \mathtt{S1,} \\ \mathtt{O2,} \\ \mathtt{S2} \end{array} \right)$

- $\mathtt{all\_coveredby(Dims, OIDS)} \stackrel{\mathrm{def}}{=}$
  $\forall \mathtt{O1} \in \mathtt{objects(OIDS)}$
    $\forall \mathtt{O2} \in \mathtt{objects(OIDS)}$
      $\mathtt{O1.oid} < \quad \Rightarrow$
      $\mathtt{O2.oid}$
      $\mathtt{coveredby\_objects} \left(\begin{array}{l} \mathtt{Dims,} \\ \mathtt{O1,} \\ \mathtt{O2} \end{array} \right)$

- $\mathtt{all\_coveredby(DIMENSIONS, OIDS)}$