5.138 elem_from_to

DESCRIPTION LINKS AUTOMATON

Origin Derived from elem.

Constraint elem_from_to(ITEM, TABLE)

Synonym element_from_to.

Arguments

$$\begin{array}{lll} \texttt{ITEM} & : & \texttt{collection}\left(\begin{array}{l} \texttt{from}-\texttt{dvar}, \\ \texttt{cst\_from}-\texttt{int}, \\ \texttt{to}-\texttt{dvar}, \\ \texttt{cst\_to}-\texttt{int}, \\ \texttt{value}-\texttt{dvar} \end{array}\right) \\ \texttt{TABLE} & : & \texttt{collection}(\texttt{index}-\texttt{int}, \texttt{value}-\texttt{dvar}) \end{array}$$

Restrictions

required(ITEM, [from, cst_from, to, cst_to, value])
ITEM.from ≥ 1
ITEM.from $\leq$ |TABLE|
ITEM.to ≥ 1
ITEM.to $\leq$ |TABLE|
ITEM.from $\leq$ ITEM.to
|ITEM| $= 1$
required(TABLE, [index, value])
TABLE.index ≥ 1
TABLE.index $\leq$ |TABLE|
increasing_seq(TABLE, [index])

Purpose

Let FROM, CST_FROM, TO, CST_TO, VALUE respectively denote the attributes ITEM[1].from, ITEM[1].cst_from, ITEM[1].to, ITEM[1].cst_to, ITEM[1].value of the unique item of the ITEM collection.
Beside imposing the fact that FROM $\leq$ TO and that both FROM and TO are assigned a value in $[1, |\texttt{TABLE}|]$, the elem_from_to constraint forces the following condition: All entries of the TABLE collection from position $\max(1, \texttt{FROM} + \texttt{CST\_FROM})$ to position $\min(|\texttt{TABLE}|, \texttt{TO} + \texttt{CST\_TO})$ are equal to VALUE. When $\max(1, \texttt{FROM} + \texttt{CST\_FROM})$ is strictly greater than $\min(|\texttt{TABLE}|, \texttt{TO} + \texttt{CST\_TO})$ the constraint holds no matter what value is assigned to VALUE.

Example

$$\left(\begin{array}{l} \langle \texttt{from}-1\ \texttt{cst\_from}-1\ \texttt{to}-4\ \texttt{cst\_to}-{-1}\ \texttt{value}-2\rangle, \\ \left\langle\begin{array}{ll} \texttt{index}-1 & \texttt{value}-6, \\ \texttt{index}-2 & \texttt{value}-2, \\ \texttt{index}-3 & \texttt{value}-2, \\ \texttt{index}-4 & \texttt{value}-9, \\ \texttt{index}-5 & \texttt{value}-9 \end{array}\right\rangle \end{array}\right)$$

The elem_from_to constraint holds since all entries between position $\max(1, \texttt{FROM} + \texttt{CST\_FROM}) = \max(1, 1+1) = 2$ and position $\min(|\texttt{TABLE}|, \texttt{TO} + \texttt{CST\_TO}) = \min(5, 4-1) = 3$ are equal to 2.

Typical

$\texttt{ITEM.cst\_from} \geq 0$
$\texttt{ITEM.cst\_from} \leq 1$
$\texttt{ITEM.cst\_to} \geq -1$
$\texttt{ITEM.cst\_to} \leq 1$
$|\texttt{TABLE}| > 1$
$\texttt{range}(\texttt{TABLE.value}) > 1$

Symmetry

All occurrences of two distinct values in ITEM.value or TABLE.value can be swapped; all occurrences of a value in ITEM.value or TABLE.value can be renamed to any unused value.

Usage

Given an array $t[1..n]$ of integers (i.e., an array of integers for which the entries are defined between 1 and n), the elem_from_to constraint is for instance useful for encoding expressions of the form $\exists i \in [1, n], \forall j \in [i+1, n] \mid t[i] = 0$. Note that, when the interval $[i+1, n]$ is empty, the condition $\forall j \in [i+1, n] \mid t[i] = 0$ is satisfied and i is equal to n. This example is encoded by using an elem_from_to constraint and by respectively setting:

- FROM to i, where i is a variable that is assigned a value from interval $[1, n]$,
- CST_FROM to constant 1,
- TO to n, the index of the last entry of the array $t[1..n]$,
- CST_TO to constant 0,
- VALUE to 0, the value we are looking for.
- TABLE to the array of integers $t[1..n]$.

Finally, note that j is not used at all.

See also

common keyword: elem, element *(array constraint)*.

Keywords

characteristic of a constraint: automaton, automaton without counters, reified automaton constraint.

constraint type: data constraint.

filtering: arc-consistency.

modelling: array constraint, table, variable indexing, variable subscript.

Automaton Figure 5.299 depicts the automaton associated with the `elem_from_to` constraint. Let us first introduce some notations:

- Let n denote the number of items of the TABLE collection.
- Let $\texttt{INDEX}_i$ and $\texttt{VALUE}_i$ respectively be the `index` and the `value` attributes of the i^{th} item of the TABLE collection.
- Let FROM, CST_FROM, TO, CST_TO, VALUE respectively denote the attributes ITEM[1].`from`, ITEM[1].`cst_from`, ITEM[1].`to`, ITEM[1].`cst_to`, ITEM[1].`value` of the unique item of the ITEM collection.
- Let IN be a shortcut for condition $1 \leq \texttt{FROM} \wedge \texttt{FROM} \leq \texttt{TO} \wedge \texttt{TO} \leq n$.
- Let F and T respectively denote the quantities $\max(1, \texttt{FROM} + \texttt{CST\_FROM})$ and $\min(|\texttt{TABLE}|, \texttt{TO} + \texttt{CST\_TO})$.

To each septuple $(\texttt{FROM}, \texttt{TO}, \texttt{F}, \texttt{T}, \texttt{VALUE}, \texttt{INDEX}_i, \texttt{VALUE}_i)$ corresponds a signature variable S_i as well as the following signature constraint:

$$\begin{cases} (\texttt{IN} \wedge \texttt{F} > \texttt{T}) & \Leftrightarrow \; S_i = 0 \wedge \\ (\texttt{IN} \wedge \texttt{F} \leq \texttt{T} \wedge \texttt{F} > \texttt{INDEX}_i) & \Leftrightarrow \; S_i = 1 \wedge \\ (\texttt{IN} \wedge \texttt{F} \leq \texttt{T} \wedge \texttt{T} < \texttt{INDEX}_i) & \Leftrightarrow \; S_i = 2 \wedge \\ (\texttt{IN} \wedge \texttt{F} \leq \texttt{T} \wedge \texttt{F} \leq \texttt{INDEX}_i \wedge \texttt{INDEX}_i \leq \texttt{T} \wedge \texttt{VALUE} = \texttt{VALUE}_i) & \Leftrightarrow \; S_i = 3 \wedge \\ (\texttt{IN} \wedge \texttt{F} \leq \texttt{T} \wedge \texttt{F} \leq \texttt{INDEX}_i \wedge \texttt{INDEX}_i \leq \texttt{T} \wedge \texttt{VALUE} \neq \texttt{VALUE}_i) & \Leftrightarrow \; S_i = 4 \end{cases} .$$

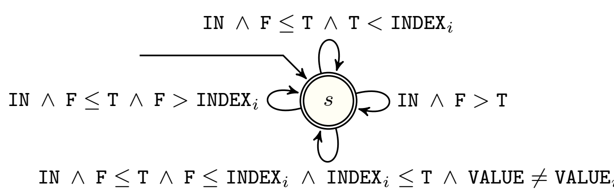

Figure 5.299: Automaton of the `elem_from_to` constraint

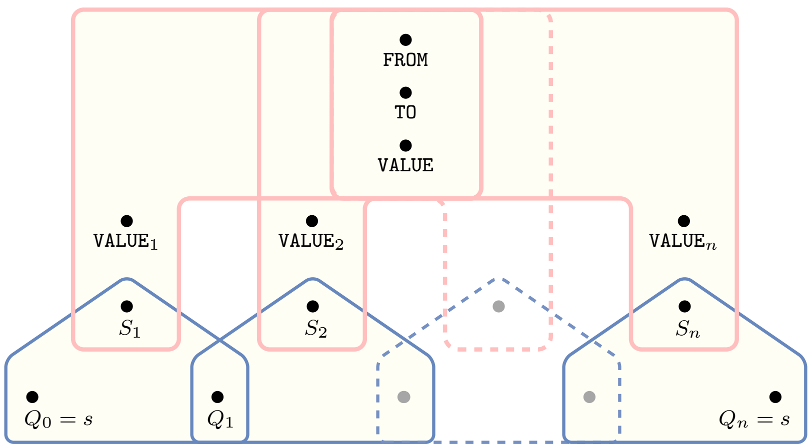

Figure 5.300: Hypergraph of the reformulation corresponding to the automaton of the `elem_from_to` constraint