

5.142 element_matrix

	DESCRIPTION	LINKS	GRAPH	AUTOMATON
Origin	CHIP			
Constraint	element_matrix(MAX_I, MAX_J, INDEX_I, INDEX_J, MATRIX, VALUE)			
Synonyms	elem_matrix, matrix.			
Arguments	<pre> MAX_I : int MAX_J : int INDEX_I : dvar INDEX_J : dvar MATRIX : collection(i-int, j-int, v-int) VALUE : dvar </pre>			
Restrictions	<pre> MAX_I ≥ 1 MAX_J ≥ 1 INDEX_I ≥ 1 INDEX_I ≤ MAX_I INDEX_J ≥ 1 INDEX_J ≤ MAX_J required(MATRIX, [i, j, v]) increasing_seq(MATRIX, [i, j]) MATRIX.i ≥ 1 MATRIX.i ≤ MAX_I MATRIX.j ≥ 1 MATRIX.j ≤ MAX_J MATRIX = MAX_I * MAX_J </pre>			
Purpose	<p>The MATRIX collection corresponds to the two-dimensional matrix MATRIX[1..MAX_I, 1..MAX_J]. VALUE is equal to the entry MATRIX[INDEX_I, INDEX_J] of the previous matrix.</p>			

Example

$$\left(4, 3, 1, 3, \left\langle \begin{array}{l} i-1 \quad j-1 \quad v-4, \\ i-1 \quad j-2 \quad v-1, \\ i-1 \quad j-3 \quad v-7, \\ i-2 \quad j-1 \quad v-1, \\ i-2 \quad j-2 \quad v-0, \\ i-2 \quad j-3 \quad v-8, \\ i-3 \quad j-1 \quad v-3, \\ i-3 \quad j-2 \quad v-2, \\ i-3 \quad j-3 \quad v-1, \\ i-4 \quad j-1 \quad v-0, \\ i-4 \quad j-2 \quad v-0, \\ i-4 \quad j-3 \quad v-6 \end{array} \right\rangle, 7 \right)$$

The `element_matrix` constraint holds since its last argument `VALUE = 7` is equal to the `v` attribute of the k^{th} item of the `MATRIX` collection such that `MATRIX[k].i = INDEX_I = 1` and `MATRIX[k].j = INDEX_J = 3`.

Typical

```
MAX_I > 1
MAX_J > 1
|MATRIX| > 3
maxval(MATRIX.i) > 1
maxval(MATRIX.j) > 1
range(MATRIX.v) > 1
```

Symmetry

All occurrences of two distinct values in `MATRIX.v` or `VALUE` can be [swapped](#); all occurrences of a value in `MATRIX.v` or `VALUE` can be [renamed](#) to any unused value.

Reformulation

The `element_matrix(MAX_I, MAX_J, INDEX_I, INDEX_J, MATRIX, VALUE)` constraint can be expressed in term of `MAX_I` `element(INDEX_J, LINEi, VARi)` ($i \in [1, \text{MAX_I}]$), where `LINEi` corresponds to the i -th line of the matrix `MATRIX` and of one `element(INDEX_I, (VAR1, VAR2, ..., VARMAX_I), VALUE)` constraint.

If we consider the **Example** slot we get the following `element` constraints:

- `element(3, (4, 1, 7), 7)`,
- `element(3, (1, 0, 8), 8)`,
- `element(3, (3, 2, 1), 1)`,
- `element(3, (0, 0, 6), 6)`,
- `element(1, (7, 8, 1, 6), 7)`.

Systems

`nth` in **Choco**, `element` in **Gecode**.

See also

common keyword: `elem`, `element` (*array constraint*).

Keywords

characteristic of a constraint: automaton, automaton without counters, reified automaton constraint, derived collection.

constraint arguments: ternary constraint.

constraint network structure: centered cyclic(3) constraint network(1).

constraint type: data constraint.

filtering: arc-consistency.

modelling: array constraint, matrix.

Derived Collection

$$\text{col} \left(\frac{\text{ITEM-collection}(\text{index}_i\text{-dvar}, \text{index}_j\text{-dvar}, \text{value}\text{-dvar}),}{[\text{item}(\text{index}_i - \text{INDEX}_I, \text{index}_j - \text{INDEX}_J, \text{value} - \text{VALUE})]} \right)$$
Arc input(s)

ITEM MATRIX

Arc generator*PRODUCT* \mapsto collection(item, matrix)**Arc arity**

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Arc constraint(s)

- item.index_i = matrix.i
- item.index_j = matrix.j
- item.value = matrix.v

Graph property(ies)NARC = 1**Graph model**

Similar to the `element` constraint except that the arc constraint is updated according to the fact that we have a two-dimensional matrix.

Parts (A) and (B) of Figure 5.311 respectively show the initial and final graph associated with the **Example** slot. Since we use the NARC graph property, the unique arc of the final graph is stressed in bold.

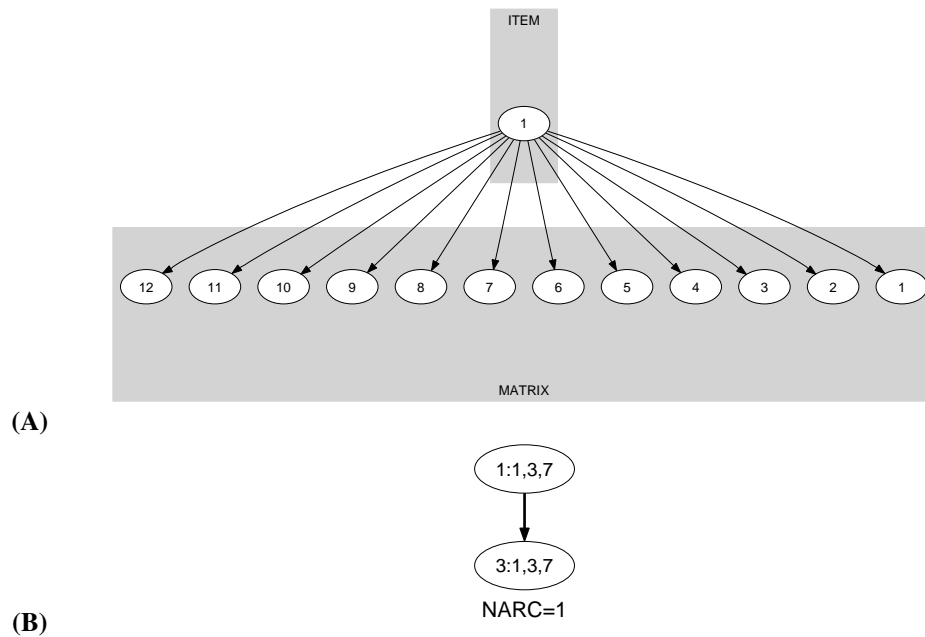


Figure 5.311: Initial and final graph of the `element_matrix` constraint

Signature

Because of the first condition of the arc constraint the final graph cannot have more than one arc. Therefore we can rewrite $\text{NARC} = 1$ to $\text{NARC} \geq 1$ and simplify NARC to NARC.

Automaton

Figure 5.312 depicts the automaton associated with the `element_matrix` constraint. Let I_k, J_k and V_k respectively be the i , the j and the v k^{th} attributes of the `MATRIX` collection. To each sextuple $(INDEX_I, INDEX_J, VALUE, I_k, J_k, V_k)$ corresponds a 0-1 signature variable S_k as well as the following signature constraint: $((INDEX_I = I_k) \wedge (INDEX_J = J_k) \wedge (VALUE = V_k)) \Leftrightarrow S_k$.

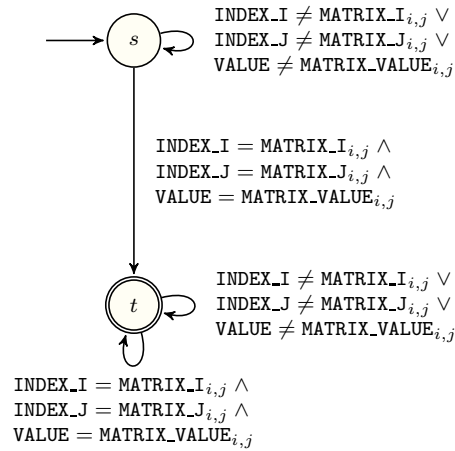


Figure 5.312: Automaton of the `element_matrix` constraint

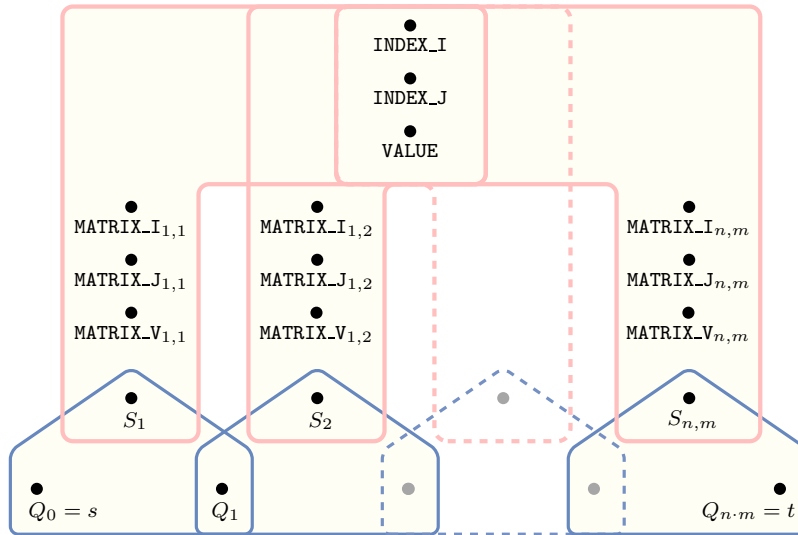


Figure 5.313: Hypergraph of the reformulation corresponding to the automaton of the `element_matrix` constraint where n and m respectively stands for `MAX_I` and `MAX_J`