

## 5.213 `k_used_by_modulo`

	DESCRIPTION	LINKS	GRAPH
<b>Origin</b>	Derived from <code>used_by_modulo</code> and from <code>k_used_by</code> .		
<b>Constraint</b>	<code>k_used_by_modulo(SETS, M)</code>		
<b>Type</b>	VARIABLES : <code>collection(var-dvar)</code>		
<b>Arguments</b>	SETS : <code>collection(set - VARIABLES)</code> M : <code>int</code>		
<b>Restrictions</b>	<code>required(VARIABLES, var)</code> $ \text{VARIABLES}  \geq 1$ <code>required(SETS, set)</code> $ \text{SETS}  > 1$ <code>non_increasing_size(SETS, set)</code> $M > 0$		
<b>Purpose</b>	Given $ \text{SETS} $ sets of domain variables, the <code>k_used_by_modulo</code> constraint forces a <code>used_by_modulo</code> constraint between each pair of consecutive sets.		
<b>Example</b>	$(\langle \text{set} - \langle 1, 9, 4, 5, 2, 1 \rangle, \text{set} - \langle 7, 1, 2, 5 \rangle, \text{set} - \langle 1, 1 \rangle \rangle, 3)$		
	The <code>k_used_by_modulo</code> constraint holds since: <ul style="list-style-type: none"> <li>• The first collection of variables is assigned 1 value in <math>\{0, 3, \dots, 3 \cdot k\}</math>, 3 values in <math>\{1, 4, \dots, 1 + 3 \cdot k\}</math> and 2 values in <math>\{2, 5, \dots, 2 + 3 \cdot k\}</math>, while the second collection of variables is assigned no more values in the previous three sets of values.</li> <li>• The second collection of variables is assigned 2 values in <math>\{0, 3, \dots, 3 \cdot k\}</math> and 2 values in <math>\{2, 5, \dots, 2 + 3 \cdot k\}</math>, while the third collection of variables is assigned no more values in the previous three sets of values.</li> </ul>		
<b>Typical</b>	$ \text{VARIABLES}  > 1$ $M > 1$		
<b>Symmetries</b>	<ul style="list-style-type: none"> <li>• Items of SETS are <code>permutable</code>.</li> <li>• Items of SETS.set are <code>permutable</code>.</li> <li>• An occurrence of a value <math>u</math> of SETS.set.var can be <code>replaced</code> by any other value <math>v</math> such that <math>v</math> is congruent to <math>u</math> modulo <math>M</math>.</li> </ul>		
<b>Arg. properties</b>	<code>Contractible</code> wrt. SETS.		

**See also**            **common keyword:** `k_used_by` (*system of constraints*).  
                  **implied by:** `k_same_modulo`.  
                  **part of system of constraints:** `used_by_modulo`.  
                  **used in graph description:** `used_by_modulo`.

**Keywords**            **characteristic of a constraint:** `modulo`, sort based reformulation.  
                  **constraint type:** `system of constraints`, `decomposition`.  
                  **modelling:** `inclusion`.

<b>Arc input(s)</b>	SETS
<b>Arc generator</b>	$\text{PATH} \mapsto \text{collection}(\text{set1}, \text{set2})$
<b>Arc arity</b>	2
<b>Arc constraint(s)</b>	$\text{used\_by\_modulo}(\text{set1.set}, \text{set2.set}, M)$
<b>Graph property(ies)</b>	$\overline{\text{NARC}} =  \text{SETS}  - 1$

**Graph model**

Parts (A) and (B) of Figure 5.467 respectively show the initial and final graph associated with the **Example** slot. To each vertex corresponds a collection of variables, while to each arc corresponds a `used_by_modulo` constraint.

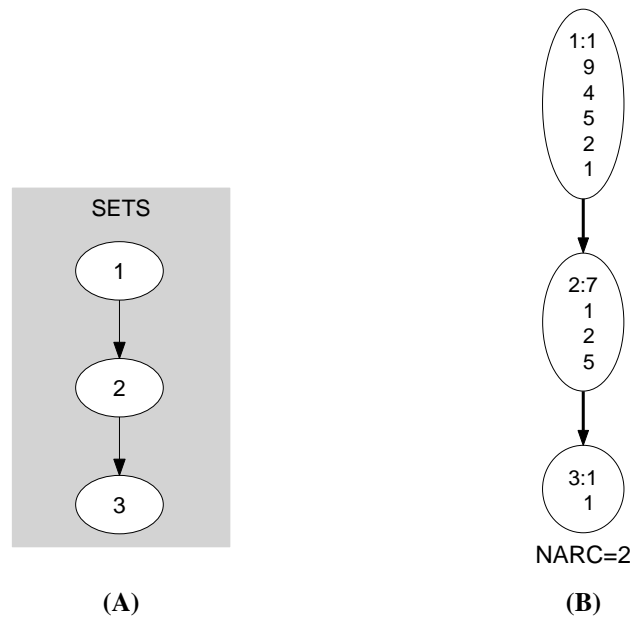


Figure 5.467: Initial and final graph of the `k_used_by_modulo` constraint

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