5.236 longest_decreasing_sequence

DESCRIPTION LINKS AUTOMATON

Origin constraint on sequences

Constraint `longest_decreasing_sequence(L, VARIABLES)`

Synonym `size_longest_decreasing_sequence`.

Arguments

L	:	dvar
VARIABLES	:	collection(var−dvar)

Restrictions

L ≥ 0
L $<$range(VARIABLES.var)
required(VARIABLES, var)

Purpose

L is the largest difference between the first and the last value of the maximum decreasing sequences of the collection VARIABLES.
A sequence of consecutive variables $X_i, X_{i+1}, \ldots, X_j$ $(1 \leq i \leq j \leq |\text{VARIABLES}|)$ of the collection of variables VARIABLES is a *maximum decreasing sequence* if all the following conditions simultaneously apply:

- $X_i \geq X_{i+1} \geq \cdots \geq X_j$,
- $i = 1$ or $X_{i-1} < X_i$,
- $i = |\text{VARIABLES}|$ or $X_j < X_{j+1}$.

Example

$(0, \langle 0, 1, 2, 5 \rangle)$
$(0, \langle 8, 8 \rangle)$
$(6, \langle 10, 8, 8, 6, 4, 9, 10, 8 \rangle)$

Figure 5.504 gives a graphical representation of the third example of the **Example** slot with its two maximum decreasing sequences in red of respective size 6 and 2. The corresponding `longest_decreasing_sequence` constraint holds since its first argument L is fixed to the maximum size 6.

Typical

L > 0
$|\text{VARIABLES}| > 1$
nval(VARIABLES.var) > 2

Symmetry One and the same constant can be added to the var attribute of all items of VARIABLES.

Arg. properties Functional dependency: L determined by VARIABLES.

Counting

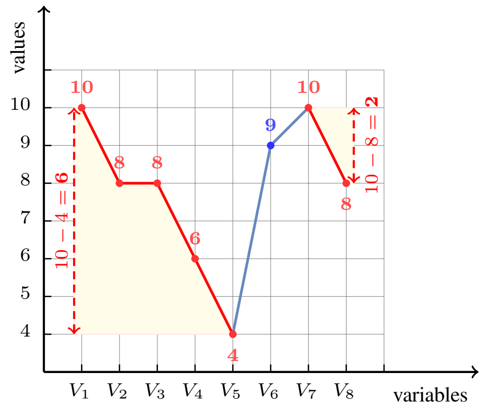
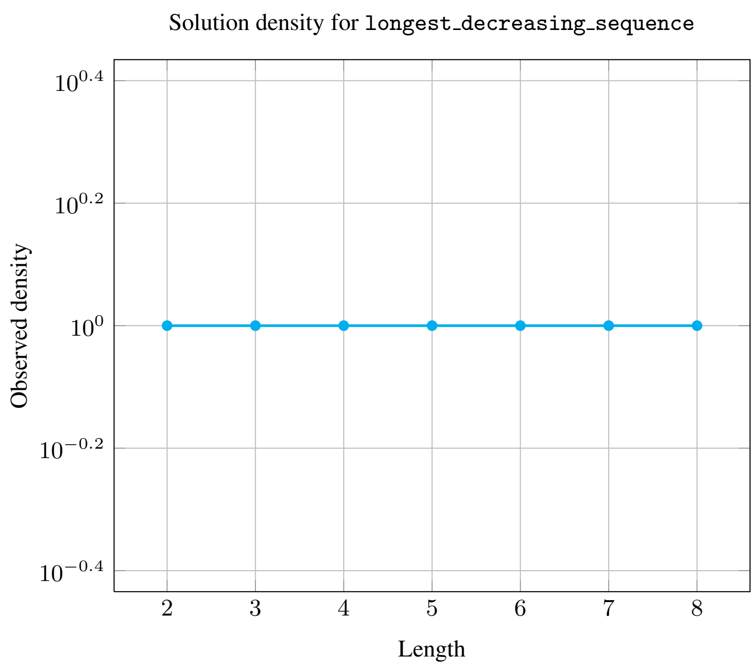
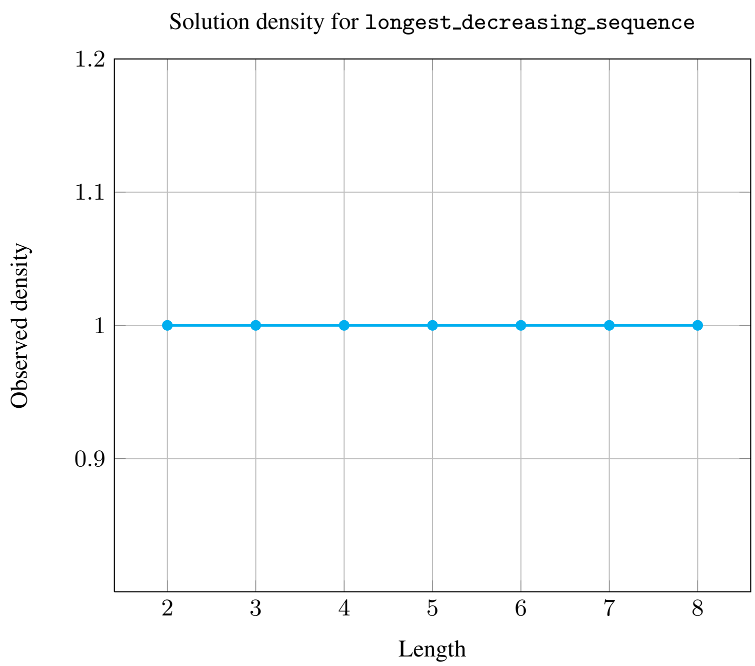


Figure 5.504: Illustration of the third example of the **Example** slot: a sequence of eight variables V_1, V_2, V_3, V_4, V_5, V_6, V_7, V_8 respectively fixed to values 10, 8, 8, 6, 4, 9, 10, 8 and its two maximum decreasing sequences in red of respective size $10 - 4 = 6$ and $10 - 8 = 2$.

Length (n)	2	3	4	5	6	7	8
Solutions	9	64	625	7776	117649	2097152	43046721

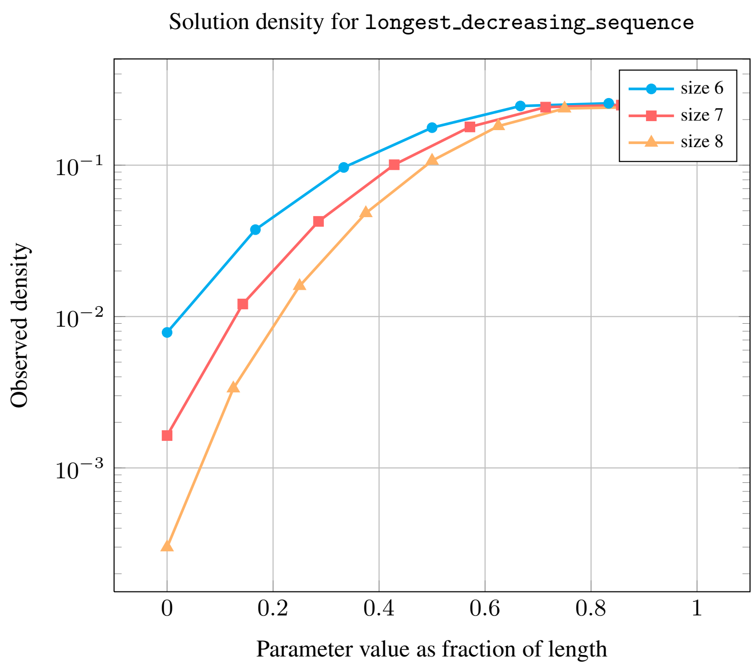
Number of solutions for `longest_decreasing_sequence`: domains $0..n$

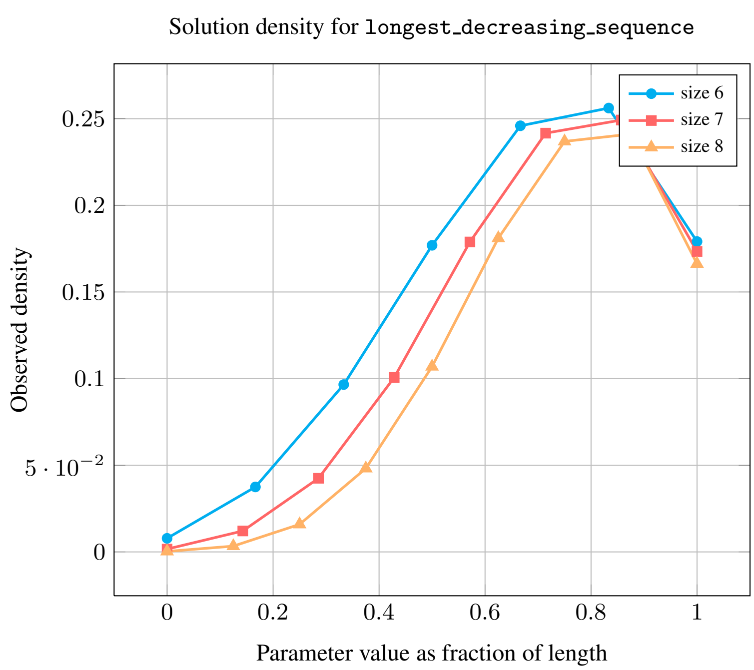




Length (n)		2	3	4	5	6	7	8
Total		9	64	625	7776	117649	2097152	43046721
Parameter value	0	6	20	70	252	924	3432	12870
	1	2	18	122	750	4412	25382	144314
	2	1	16	161	1398	11361	89132	685090
	3	-	10	162	1942	20816	211106	2074365
	4	-	-	110	2024	28930	375084	4603682
	5	-	-	-	1410	30134	506766	7792840
	6	-	-	-	-	21072	522648	10197174
	7	-	-	-	-	-	363602	10379696
	8	-	-	-	-	-	-	7156690

Solution count for longest_decreasing_sequence: domains $0..n$





See also **common keyword:** `longest_increasing_sequence`, `min_dist_between_inflexion` *(sequence)*.

Keywords **characteristic of a constraint:** automaton, automaton with counters, automaton with same input symbol.

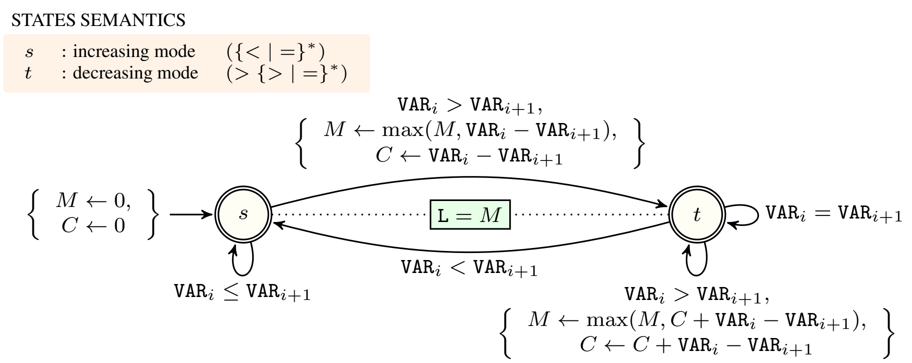
combinatorial object: sequence.

constraint arguments: reverse of a constraint, pure functional dependency.

filtering: glue matrix.

modelling: functional dependency.

Automaton Figure 5.505 depicts the automaton associated with the `longest_decreasing_sequence` constraint.



Glue matrix where $\overrightarrow{M}$, $\overrightarrow{C}$ and $\overleftarrow{M}$, $\overleftarrow{C}$ resp. represent the counters values M, C at the end of a prefix and at the end of the corresponding reverse suffix that partitions the sequence `VARIABLES`.

	s $(\{>\mid=\}^*)$	t $(<\{<\mid=\}^*)$
s $(\{<\mid=\}^*)$	$\max(\overrightarrow{M},\overleftarrow{M})$	$\max(\overrightarrow{M},\overleftarrow{M})$
t $(>\{>\mid=\}^*)$	$\max(\overrightarrow{M},\overleftarrow{M})$	$\max(\overrightarrow{M},\overrightarrow{C}+\overleftarrow{C},\overleftarrow{M})$

Figure 5.505: Automaton of the `longest_decreasing_sequence` constraint and its glue matrix (note that the reverse of the `longest_decreasing_sequence` constraint is the `longest_increasing_sequence` constraint)

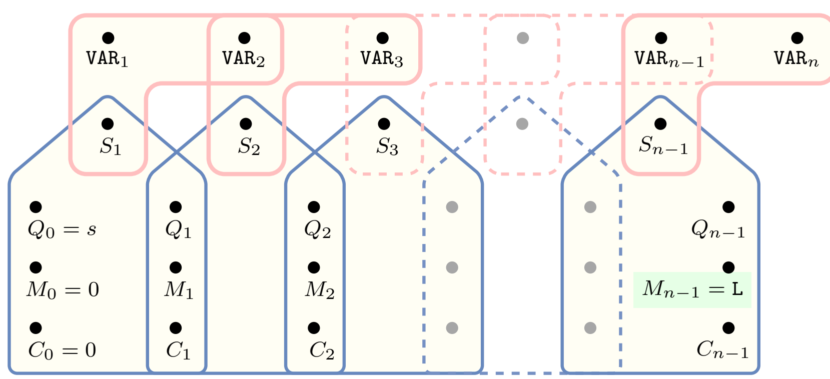


Figure 5.506: Hypergraph of the reformulation corresponding to the automaton of the `longest_decreasing_sequence` constraint