## 5.241 max_increasing_slope

DESCRIPTION LINKS AUTOMATON

**Origin**

Motivated by time series.

**Constraint**

max_increasing_slope(MAX, VARIABLES)

**Arguments**

MAX : dvar
VARIABLES : collection(var−dvar)

**Restrictions**

$\texttt{MAX} \geq 0$
$\texttt{MAX} <$ range(VARIABLES.var)
required(VARIABLES, var)
$|\texttt{VARIABLES}| > 0$

**Purpose**

Given a sequence of variables VARIABLES $= V_1, V_2, \ldots, V_n$, sets MAX to 0 if $\nexists i \in [1, n-1] | V_i < V_{i+1}$, otherwise sets MAX to $\max_{i \in [1,n-1] | V_i < V_{i+1}} (V_{i+1} - V_i)$.

**Example**

$(4, \langle 1, 1, 5, 8, 6, 2, 2, 1, 2 \rangle)$
$(0, \langle 9, 8, 6, 4, 1, 0 \rangle)$
$(8, \langle 9, 6, 6, 4, 1, 9 \rangle)$

The first max_increasing_slope constraint holds since the sequence 1 1 5 8 6 2 2 1 2 contains two increasing subsequences 1 5 8 and 1 2 and the maximum slope is equal to $\max(5-1, 8-5, 2-1) = 4$ as shown on Figure 5.513.

**Typical**

$\texttt{MAX} > 0$
$\texttt{MAX} <$ range(VARIABLES.var) $- 1$
$|\texttt{VARIABLES}| > 2$
range(VARIABLES.var) $> 2$

**Symmetry**

One and the same constant can be added to the var attribute of all items of VARIABLES.

**Arg. properties**

Functional dependency: MAX determined by VARIABLES.

**Usage**

Getting the maximum slope over the increasing sequences of time series.

**Counting**

| Length ($n$) | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|
| Solutions | 9 | 64 | 625 | 7776 | 117649 | 2097152 | 43046721 |

Number of solutions for max_increasing_slope: domains $0..n$

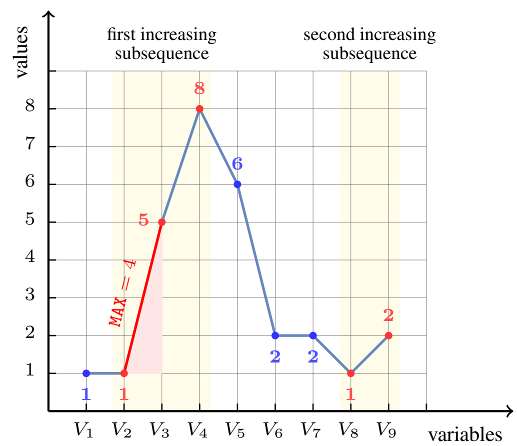
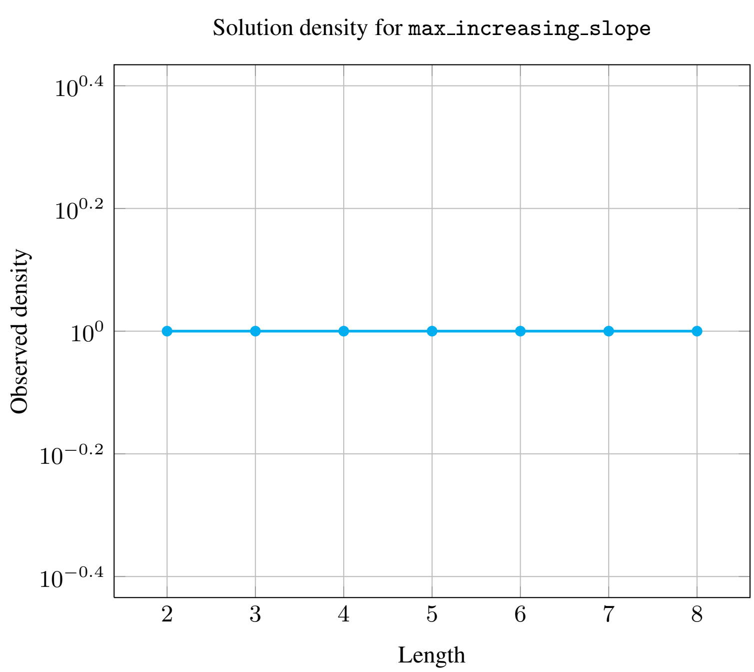
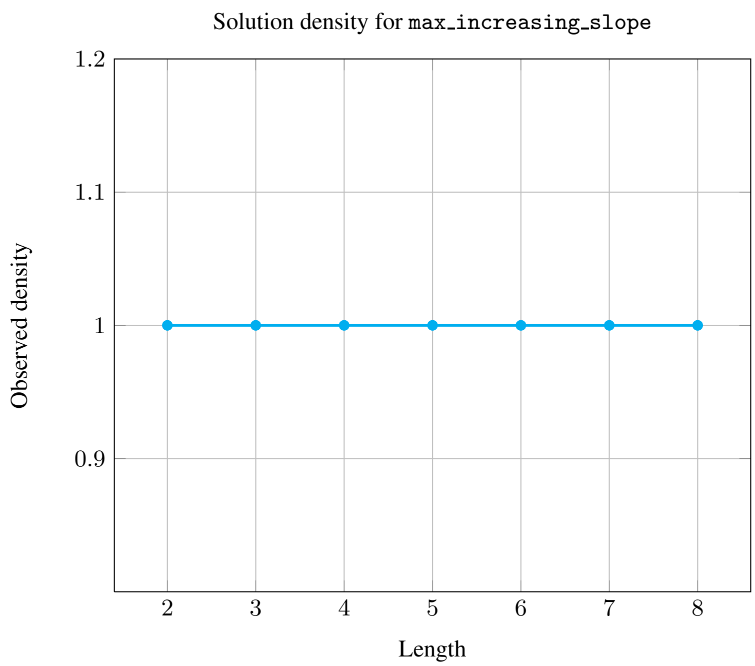


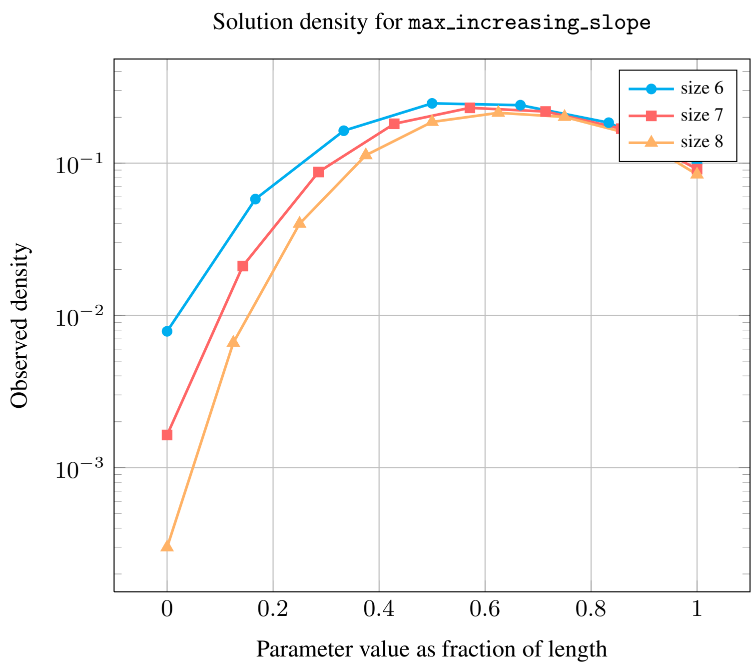

Figure 5.513: Illustration of the first example of the **Example** slot: a sequence of nine variables $V_1$, $V_2$, $V_3$, $V_4$, $V_5$, $V_6$, $V_7$, $V_8$, $V_9$ respectively fixed to values 1, 1, 5, 8, 6, 2, 2, 1, 2 and the corresponding maximum slope on the strictly increasing subsequences 1 5 8 and 1 2 ($\mathtt{MAX} = 4$)

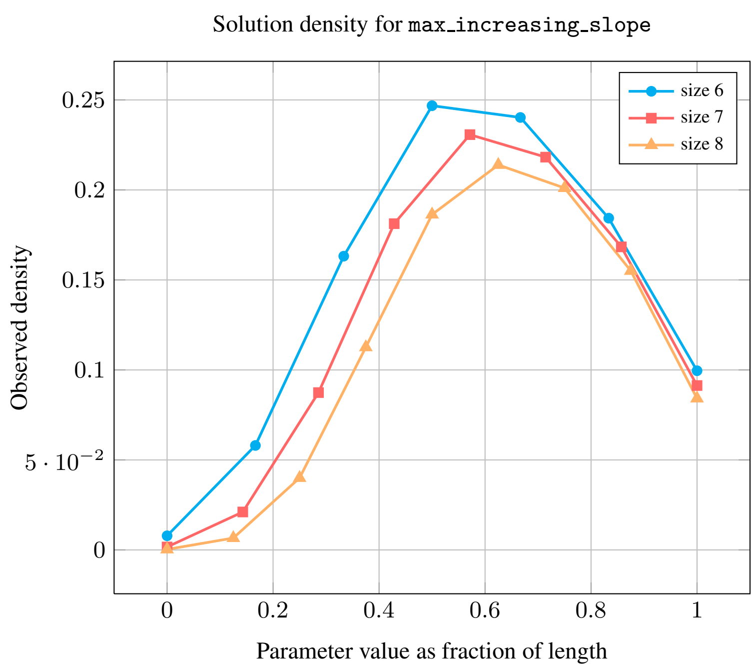





| Length ($n$) | | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Total | | 9 | 64 | 625 | 7776 | 117649 | 2097152 | 43046721 |
| Parameter value | 0 | 6 | 20 | 70 | 252 | 924 | 3432 | 12870 |
| | 1 | 2 | 20 | 151 | 1036 | 6828 | 44220 | 284405 |
| | 2 | 1 | 16 | 188 | 1952 | 19200 | 183304 | 1721425 |
| | 3 | - | 8 | 142 | 2106 | 29035 | 380116 | 4847301 |
| | 4 | - | - | 74 | 1584 | 28266 | 483840 | 8021350 |
| | 5 | - | - | - | 846 | 21684 | 457632 | 9208124 |
| | 6 | - | - | - | - | 11712 | 353088 | 8654931 |
| | 7 | - | - | - | - | - | 191520 | 6673834 |
| | 8 | - | - | - | - | - | - | 3622481 |

Solution count for max_increasing_slope: domains $0..n$




**Keywords** **characteristic of a constraint:** automaton, automaton with counters.

**combinatorial object:** sequence.

**constraint arguments:** reverse of a constraint, pure functional dependency.

**filtering:** glue matrix.

**modelling:** functional dependency.

**Cond. implications**

- max_increasing_slope(MAX, VARIABLES)
  with range(VARIABLES.var) = MAX + 1
  **implies** longest_increasing_sequence(L, VARIABLES)
  when range(VARIABLES.var) = L + 1.

- max_increasing_slope(MAX, VARIABLES)
  with MAX = 1
  **implies** min_increasing_slope(MIN, VARIABLES)
  when MIN = 1.

**Automaton** Figure 5.514 depicts the automaton associated with the `max_increasing_slope` constraint. To each pair of consecutive variables $(\texttt{VAR}_i, \texttt{VAR}_{i+1})$ of the collection `VARIABLES` corresponds a signature variable $S_i$. The following signature constraint links $\texttt{VAR}_i$, $\texttt{VAR}_{i+1}$ and $S_i$: $(\texttt{VAR}_i \geq \texttt{VAR}_{i+1} \Leftrightarrow S_i = 0) \wedge (\texttt{VAR}_i < \texttt{VAR}_{i+1} \Leftrightarrow S_i = 1)$.

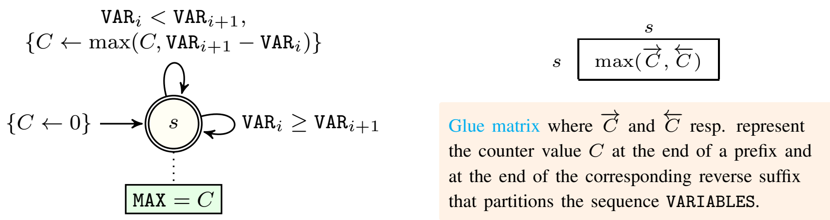



Figure 5.514: Automaton for the `max_increasing_slope` constraint and its glue matrix (note that the reverse of `max_increasing_slope` is `max_decreasing_slope`)