5.264 minimum_greater_than

DESCRIPTION LINKS GRAPH AUTOMATON

Origin N. Beldiceanu

Constraint minimum_greater_than(VAR1, VAR2, VARIABLES)

Arguments

VAR1 : dvar
VAR2 : dvar
VARIABLES : collection(var−dvar)

Restrictions

VAR1 > VAR2
|VARIABLES| > 0
required(VARIABLES, var)

Purpose VAR1 is the smallest value strictly greater than VAR2 of the collection of variables VARIABLES: this concretely means that there exists at least one variable of VARIABLES that takes a value strictly greater than VAR2.

Example $(5, 3, \langle 8, 5, 3, 8\rangle)$

The minimum_greater_than constraint holds since value 5 is the smallest value strictly greater than value 3 among values 8, 5, 3 and 8.

Typical

|VARIABLES| > 1
range(VARIABLES.var) > 1

Symmetry Items of VARIABLES are permutable.

Arg. properties Aggregate: VAR1(min), VAR2(id), VARIABLES(union).

Reformulation Let $V_1, V_2, \ldots, V_{|\text{VARIABLES}|}$ denote the variables of the collection of variables VARIABLES. By creating the extra variables M and $U_1, U_2, \ldots, U_{|\text{VARIABLES}|}$, the minimum_greater_than constraint can be expressed in term of the following constraints:

1. maximum(M, VARIABLES),
2. VAR1 > VAR2,
3. VAR1 $\leq M$,
4. $V_i \leq$ VAR2 $\Rightarrow U_i = M$ $(i \in [1, |\text{VARIABLES}|])$,
5. $V_i >$ VAR2 $\Rightarrow U_i = V_i$ $(i \in [1, |\text{VARIABLES}|])$,
6. minimum(VAR1, $\langle U_1, U_2, \ldots, U_{|\text{VARIABLES}|}\rangle$).

See also

common keyword: next_greater_element (*order constraint*).

implied by: next_greater_element.

related: next_element (*identify an element in a table*).

Keywords

characteristic of a constraint: minimum, automaton, automaton without counters, reified automaton constraint, derived collection.

constraint network structure: centered cyclic(2) constraint network(1).

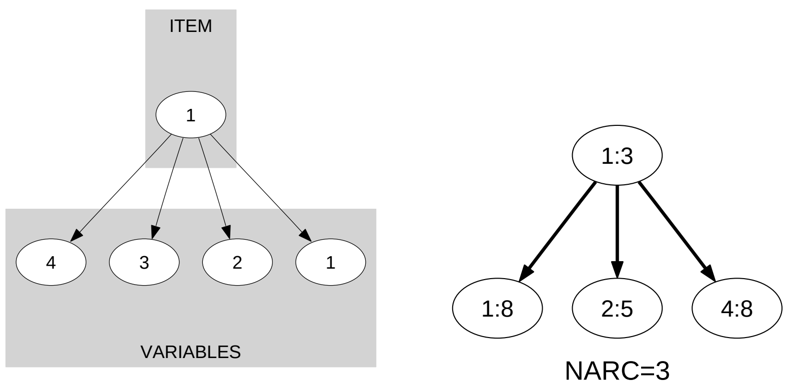
constraint type: order constraint.

Derived Collection	`col(ITEM−collection(var−dvar), [item(var − VAR2)])`
Arc input(s)	ITEM VARIABLES
Arc generator	$PRODUCT$ ↦ `collection(item, variables)`
Arc arity	2
Arc constraint(s)	`item.var` < `variables.var`
Graph property(ies)	$\mathbf{NARC} > 0$
Sets	SUCC ↦ [`source`, `variables`]
Constraint(s) on sets	`minimum(VAR1, variables)`

Graph model

Similar to the `next_greater_element` constraint, except that there is no order on the variables of the collection VARIABLES.

Parts (A) and (B) of Figure 5.565 respectively show the initial and final graph associated with the **Example** slot. Since we use the $\mathbf{NARC}$ graph property, the arcs of the final graph are stressed in bold. The source and the sinks of the final graph respectively correspond to the variable VAR2 and to the variables of the VARIABLES collection that are strictly greater than VAR2. VAR1 is set to the smallest value of the var attribute of the sinks of the final graph.



(A) **(B)**

Figure 5.565: Initial and final graph of the `minimum_greater_than` constraint

Automaton Figure 5.566 depicts the automaton associated with the minimum_greater_than constraint. Let $\texttt{VAR}_i$ be the i^{th} variable of the VARIABLES collection. To each triple $(\texttt{VAR1}, \texttt{VAR2}, \texttt{VAR}_i)$ corresponds a signature variable S_i as well as the following signature constraint:

$$((\texttt{VAR}_i < \texttt{VAR1}) \wedge (\texttt{VAR}_i \leq \texttt{VAR2})) \Leftrightarrow S_i = 0 \wedge$$
$$((\texttt{VAR}_i = \texttt{VAR1}) \wedge (\texttt{VAR}_i \leq \texttt{VAR2})) \Leftrightarrow S_i = 1 \wedge$$
$$((\texttt{VAR}_i > \texttt{VAR1}) \wedge (\texttt{VAR}_i \leq \texttt{VAR2})) \Leftrightarrow S_i = 2 \wedge$$
$$((\texttt{VAR}_i < \texttt{VAR1}) \wedge (\texttt{VAR}_i > \texttt{VAR2})) \Leftrightarrow S_i = 3 \wedge$$
$$((\texttt{VAR}_i = \texttt{VAR1}) \wedge (\texttt{VAR}_i > \texttt{VAR2})) \Leftrightarrow S_i = 4 \wedge$$
$$((\texttt{VAR}_i > \texttt{VAR1}) \wedge (\texttt{VAR}_i > \texttt{VAR2})) \Leftrightarrow S_i = 5.$$

The automaton is constructed in order to fulfil the following conditions:

- We look for an item of the VARIABLES collection such that $\texttt{var}_i = \texttt{VAR1}$ and $\texttt{var}_i > \texttt{VAR2}$,
- There should not exist any item of the VARIABLES collection such that $\texttt{var}_i < \texttt{VAR1}$ and $\texttt{var}_i > \texttt{VAR2}$.

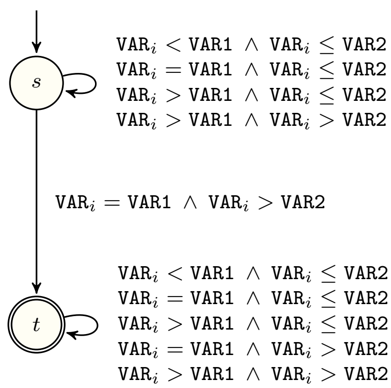


Figure 5.566: Automaton of the minimum_greater_than constraint

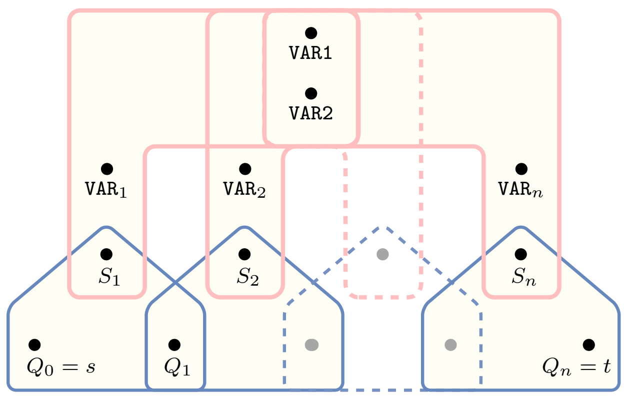


Figure 5.567: Hypergraph of the reformulation corresponding to the counter free non deterministic automaton of the `minimum_greater_than` constraint