5.306 ordered_atleast_nvector

DESCRIPTION LINKS GRAPH

Origin Conjoin atleast_nvector and lex_chain_lesseq.

Constraint ordered_atleast_nvector(NVEC, VECTORS)

Synonyms ordered_atleast_nvectors, ordered_atleast_npoint, ordered_atleast_npoints.

Type VECTOR : collection(var−dvar)

Arguments

NVEC	:	dvar
VECTORS	:	collection(vec − VECTOR)

Restrictions

$|\text{VECTOR}| \geq 1$
$\text{NVEC} \geq 0$
$\text{NVEC} \leq |\text{VECTORS}|$
required(VECTORS, vec)
same_size(VECTORS, vec)

Purpose

Enforces the following two conditions:

1. The number of distinct tuples of values taken by the vectors of the collection VECTORS is greater than or equal to NVEC. Two tuples of values $\langle A_1, A_2, \ldots, A_m \rangle$ and $\langle B_1, B_2, \ldots, B_m \rangle$ are *distinct* if and only if there exist an integer $i \in [1, m]$ such that $A_i \neq B_i$.
2. For each pair of consecutive vectors VECTOR_i and VECTOR_{i+1} of the VECTORS collection we have that VECTOR_i is lexicographically less than or equal to VECTOR_{i+1}. Given two vectors, $\vec{X}$ and $\vec{Y}$ of n components, $\langle X_0, \ldots, X_{n-1} \rangle$ and $\langle Y_0, \ldots, Y_{n-1} \rangle$, $\vec{X}$ is *lexicographically less than or equal to* $\vec{Y}$ if and only if $n = 0$ or $X_0 < Y_0$ or $X_0 = Y_0$ and $\langle X_1, \ldots, X_{n-1} \rangle$ is lexicographically less than or equal to $\langle Y_1, \ldots, Y_{n-1} \rangle$.

Example

$$\left(\begin{array}{c} \text{vec} - \langle 5, 6 \rangle, \\ 2, \left\langle \begin{array}{c} \text{vec} - \langle 5, 6 \rangle, \\ \text{vec} - \langle 5, 6 \rangle, \\ \text{vec} - \langle 9, 3 \rangle, \end{array} \right\rangle \\ \text{vec} - \langle 9, 4 \rangle \end{array} \right)$$

The ordered_atleast_nvector constraint holds since:

1. The collection VECTORS involves at least 2 distinct tuples of values (i.e., in fact the 3 distinct tuples $\langle 5, 6 \rangle$, $\langle 9, 3 \rangle$ and $\langle 9, 4 \rangle$).
2. The vectors of the collection VECTORS are sorted in increasing lexicographical order.

Typical

$|\texttt{VECTOR}| > 1$
$\texttt{NVEC} > 0$
$\texttt{NVEC} < |\texttt{VECTORS}|$
$|\texttt{VECTORS}| > 1$

Symmetry

NVEC can be decreased to any value ≥ 0.

Reformulation

The ordered_atleast_nvector constraint can be reformulated as a conjunction of a atleast_nvector and a lex_chain_lesseq constraints.

See also

common keyword: nvector *(vector)*.

comparison swapped: ordered_atmost_nvector.

implied by: ordered_nvector *($\geq$ NVEC replaced by $=$ NVEC)*.

implies: atleast_nvector, lex_chain_lesseq *(NVEC of constraint ordered_atleast_nvector removed)*.

used in graph description: lex_less, lex_lesseq.

Keywords

characteristic of a constraint: vector.

constraint type: counting constraint, order constraint.

symmetry: symmetry.

Arc input(s)	VECTORS
Arc generator	$PATH \mapsto$ collection(vectors1, vectors2)
Arc arity	2
Arc constraint(s)	lex_lesseq(vectors1.vec, vectors2.vec)
Graph property(ies)	$\mathbf{NARC} = \lvert\text{VECTORS}\rvert - 1$

Arc input(s)	VECTORS
Arc generator	$PATH \mapsto$ collection(vectors1, vectors2)
Arc arity	2
Arc constraint(s)	lex_less(vectors1.vec, vectors2.vec)
Graph property(ies)	$\mathbf{NCC} \geq$ NVEC

Graph model Parts (A) and (B) of Figure 5.626 respectively show the initial and final graph of the second graph constraint associated with the **Example** slot. Since we use the **NCC** graph property in this second graph constraint, we show the different connected components of the final graph. Each strongly connected component corresponds to a tuple of values that is assigned to some vectors of the VECTORS collection. The 3 following tuple of values $\langle 5, 6\rangle$, $\langle 9, 3\rangle$ and $\langle 9, 4\rangle$ are used by the vectors of the VECTORS collection.

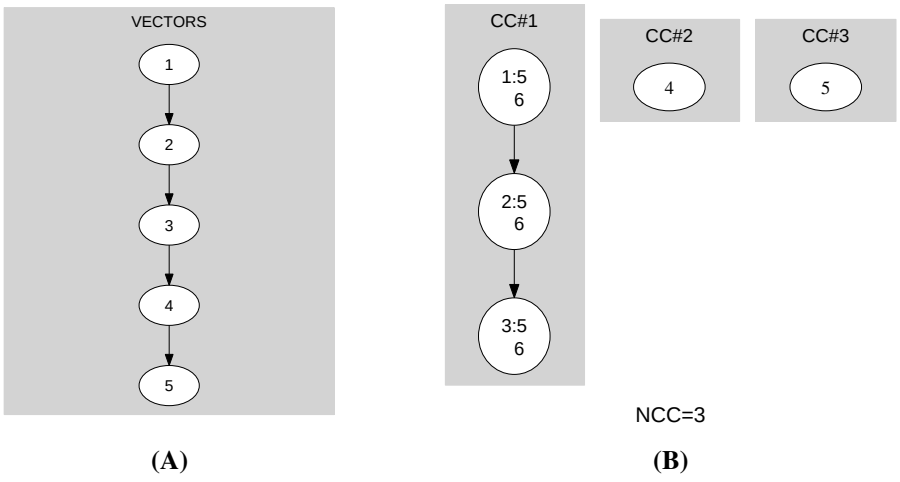

Figure 5.626: Initial and final graph of the ordered_atleast_nvector constraint