5.336 same_and_global_cardinality_low_up

DESCRIPTION LINKS GRAPH

Origin Derived from same and global_cardinality_low_up

Constraint same_and_global_cardinality_low_up(VARIABLES1, VARIABLES2, VALUES)

Arguments

VARIABLES1	:	collection(var−dvar)
VARIABLES2	:	collection(var−dvar)
VALUES	:	collection(val−int, omin−int, omax−int)

Restrictions

$|\texttt{VARIABLES1}| = |\texttt{VARIABLES2}|$
required(VARIABLES1, var)
required(VARIABLES2, var)
required(VALUES, [val, omin, omax])
distinct(VALUES, val)
$\texttt{VALUES.omin} \geq 0$
$\texttt{VALUES.omax} \leq |\texttt{VARIABLES1}|$
$\texttt{VALUES.omin} \leq \texttt{VALUES.omax}$

Purpose The variables of the VARIABLES2 collection correspond to the variables of the VARIABLES1 collection according to a permutation. In addition, each value $\texttt{VALUES}[i].\texttt{val}$ (with $i \in [1, |\texttt{VALUES}|]$) should be taken by at least $\texttt{VALUES}[i].\texttt{omin}$ and at most $\texttt{VALUES}[i].\texttt{omax}$ variables of the VARIABLES1 collection. Finally, each variable of VARIABLES1 should be assigned a value of $\texttt{VALUES}[i].\texttt{val}$ (with $i \in [1, |\texttt{VALUES}|]$).

Example

$$\left(\begin{array}{l} \langle 1, 9, 1, 5, 2, 1\rangle, \\ \langle 9, 1, 1, 1, 2, 5\rangle, \\ \left\langle \begin{array}{lll} \texttt{val}-1 & \texttt{omin}-2 & \texttt{omax}-3, \\ \texttt{val}-2 & \texttt{omin}-1 & \texttt{omax}-1, \\ \texttt{val}-5 & \texttt{omin}-1 & \texttt{omax}-1, \\ \texttt{val}-7 & \texttt{omin}-0 & \texttt{omax}-2, \\ \texttt{val}-9 & \texttt{omin}-1 & \texttt{omax}-1 \end{array} \right\rangle \end{array}\right)$$

The same_and_global_cardinality_low_up constraint holds since:

- The values 1, 9, 1, 5, 2, 1 assigned to |VARIABLES1| correspond to a permutation of the values 9, 1, 1, 1, 2, 5 assigned to |VARIABLES2|.
- The values 1, 2, 5, 7 and 6 are respectively used 3 ($2 \leq 3 \leq 3$), 1 ($1 \leq 1 \leq 1$), 1 ($1 \leq 1 \leq 1$), 0 ($0 \leq 0 \leq 2$) and 1 ($1 \leq 1 \leq 1$) times.

Typical

$|\texttt{VARIABLES1}| > 1$
$\texttt{range}(\texttt{VARIABLES1.var}) > 1$
$\texttt{range}(\texttt{VARIABLES2.var}) > 1$
$|\texttt{VALUES}| > 1$
$\texttt{VALUES.omin} \leq |\texttt{VARIABLES1}|$
$\texttt{VALUES.omax} > 0$
$\texttt{VALUES.omax} < |\texttt{VARIABLES1}|$
$|\texttt{VARIABLES1}| > |\texttt{VALUES}|$

Symmetries

- Arguments are permutable w.r.t. permutation $(\texttt{VARIABLES1}, \texttt{VARIABLES2})$ $(\texttt{VALUES})$.
- Items of VARIABLES1 are permutable.
- Items of VARIABLES2 are permutable.
- An occurrence of a value of VARIABLES1.var or VARIABLES2.var that does not belong to VALUES.val can be replaced by any other value that also does not belong to VALUES.val.
- Items of VALUES are permutable.
- VALUES.omin can be decreased to any value ≥ 0.
- VALUES.omax can be increased to any value $\leq |\texttt{VARIABLES1}|$.
- All occurrences of two distinct values in VARIABLES1.var, VARIABLES2.var or VALUES.val can be swapped; all occurrences of a value in VARIABLES1.var, VARIABLES2.var or VALUES.val can be renamed to any unused value.

Arg. properties

Contractible wrt. VALUES.

Usage

The same_and_global_cardinality_low_up constraint can be used for modelling the following assignment problem with a single constraint. The organisation Doctors Without Borders has a list of doctors and a list of nurses, each of whom volunteered to go on one rescue mission. Each volunteer specifies a list of possible dates and each mission should include one doctor and one nurse. In addition we have for each date the minimum and maximum number of missions that should be effectively done. The task is to produce a list of pairs such that each pair includes a doctor and a nurse who are available on the same date and each volunteer appears in exactly one pair so that for each day we build the required number of missions.

Algorithm

In [50], the flow network that was used to model the same constraint [47, 48] is extended to support the cardinalities. Figure 3.32 illustrates this flow model. Then, algorithms are developed to compute arc-consistency and bound-consistency.

See also

generalisation: same_and_global_cardinality *(*fixed interval *replaced by* variable*)*.

implies: global_cardinality_low_up, global_cardinality_low_up_no_loop, same.

Keywords

application area: assignment.

combinatorial object: permutation, multiset.

constraint arguments: constraint between two collections of variables.

2032$\underline{\overline{\mathbf{NSINK}}}$, $\underline{\overline{\mathbf{NSOURCE}}}$, CC($\underline{\overline{\mathbf{NSINK}}}$, $\underline{\overline{\mathbf{NSOURCE}}}$), $PRODUCT$; $\underline{\overline{\mathbf{NVERTEX}}}$, $SELF$, $\forall$

constraint type: value constraint.

filtering: bound-consistency, arc-consistency, flow.

modelling: equality between multisets.

problems: demand profile.

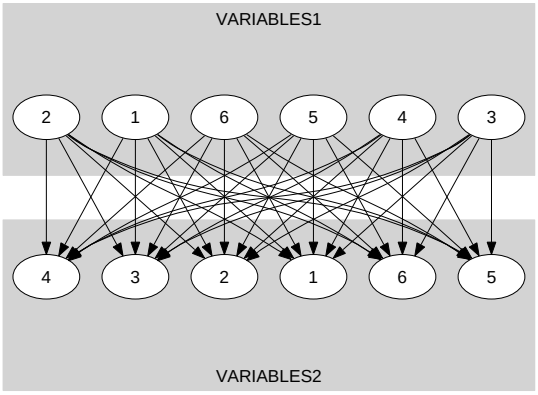
Arc input(s)	VARIABLES1 VARIABLES2
Arc generator	$PRODUCT \mapsto$ collection(variables1, variables2)
Arc arity	2
Arc constraint(s)	variables1.var = variables2.var
Graph property(ies)	• for all connected components: **NSOURCE**=**NSINK** • **NSOURCE**= |VARIABLES1| • **NSINK**= |VARIABLES2|

For all items of VALUES:

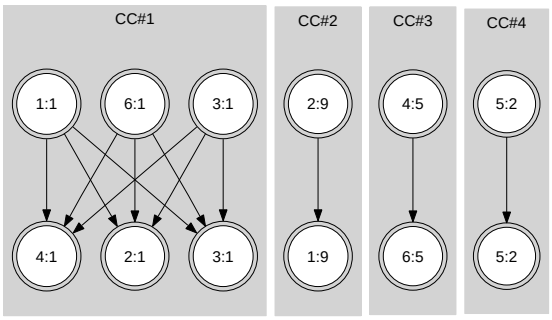
Arc input(s)	VARIABLES1
Arc generator	$SELF \mapsto$ collection(variables)
Arc arity	1
Arc constraint(s)	variables.var = VALUES.val
Graph property(ies)	• **NVERTEX**$\geq$ VALUES.omin • **NVERTEX**$\leq$ VALUES.omax

Graph model Parts (A) and (B) of Figure 5.669 respectively show the initial and final graph associated with the first graph constraint of the **Example** slot. Since we use the **NSOURCE** and **NSINK** graph properties, the source and sink vertices of the final graph are stressed with a double circle. Since there is a constraint on each connected component of the final graph we also show the different connected components. Each of them corresponds to an equivalence class according to the arc constraint.

2034$\overline{\mathbf{NSINK}}$, $\overline{\mathbf{NSOURCE}}$, CC($\overline{\mathbf{NSINK}}$, $\overline{\mathbf{NSOURCE}}$), $PRODUCT$; $\overline{\mathbf{NVERTEX}}$, $SELF$, $\forall$



(A)



CC#1:NSOURCE=3,NSINK=3
CC#2:NSOURCE=1,NSINK=1
CC#3:NSOURCE=1,NSINK=1
CC#4:NSOURCE=1,NSINK=1

(B)

Figure 5.669: Initial and final graph of the same_and_global_cardinality_low_up constraint