5.376 stretch_path

DESCRIPTION LINKS GRAPH AUTOMATON

Origin [305]

Constraint stretch_path(VARIABLES, VALUES)

Usual name stretch

Arguments

VARIABLES : collection(var−dvar)
VALUES : collection(val−int, lmin−int, lmax−int)

Restrictions

|VARIABLES| > 0
required(VARIABLES, var)
|VALUES| > 0
required(VALUES, [val, lmin, lmax])
distinct(VALUES, val)
VALUES.lmin ≥ 0
VALUES.lmin ≤ VALUES.lmax
VALUES.lmin ≤ |VARIABLES|

Purpose

In order to define the meaning of the stretch_path constraint, we first introduce the notions of *stretch* and *span*. Let n be the number of variables of the collection VARIABLES. Let $X_i, \dots, X_j$ $(1 \leq i \leq j \leq n)$ be consecutive variables of the collection of variables VARIABLES such that the following conditions apply:

- All variables $X_i, \dots, X_j$ take a same value from the set of values of the val attribute,
- $i = 1$ or X_{i-1} is different from X_i,
- $j = n$ or X_{j+1} is different from X_j.

We call such a set of variables a *stretch*. The *span* of the stretch is equal to $j - i + 1$, while the *value* of the stretch is X_i. We now define the condition enforced by the stretch_path constraint.

Each item $(\texttt{val} - v, \texttt{lmin} - s, \texttt{lmax} - t)$ of the VALUES collection enforces the minimum value s as well as the maximum value t for the span of a stretch of value v over consecutive variables of the VARIABLES collection.

Note that:

1. Having an item $(\texttt{val} - v, \texttt{lmin} - s, \texttt{lmax} - t)$ with s strictly greater than 0 does not mean that value v should be assigned to one of the variables of collection VARIABLES. It rather means that, when value v is used, all stretches of value v must have a span that belong to interval $[s, t]$.
2. A variable of the collection VARIABLES may be assigned a value that is not defined in the VALUES collection.

Example

$$\left(\begin{array}{l}\langle 6,6,3,1,1,1,6,6\rangle, \\ \left\langle\begin{array}{lll}\texttt{val}-1 & \texttt{lmin}-2 & \texttt{lmax}-4, \\ \texttt{val}-2 & \texttt{lmin}-2 & \texttt{lmax}-3, \\ \texttt{val}-3 & \texttt{lmin}-1 & \texttt{lmax}-6, \\ \texttt{val}-6 & \texttt{lmin}-2 & \texttt{lmax}-2\end{array}\right\rangle\end{array}\right)$$

The `stretch_path` constraint holds since the sequence 6 6 3 1 1 1 6 6 contains four stretches 6 6, 3, 1 1 1, and 6 6 respectively verifying the following conditions:

- The span of the first stretch 6 6 is located within interval $[2, 2]$ (i.e., the limit associated with value 6).
- The span of the second stretch 3 is located within interval $[1, 6]$ (i.e., the limit associated with value 3).
- The span of the third stretch 1 1 1 is located within interval $[2, 4]$ (i.e., the limit associated with value 1).
- The span of the fourth stretch 6 6 is located within interval $[2, 2]$ (i.e., the limit associated with value 6).

Typical

$|\texttt{VARIABLES}| > 1$
$\texttt{range}(\texttt{VARIABLES.var}) > 1$
$|\texttt{VARIABLES}| > |\texttt{VALUES}|$
$|\texttt{VALUES}| > 1$
$\texttt{sum}(\texttt{VALUES.lmin}) \leq |\texttt{VARIABLES}|$
$\texttt{VALUES.lmax} \leq |\texttt{VARIABLES}|$

Symmetries

- Items of `VARIABLES` can be reversed.
- Items of `VALUES` are permutable.
- All occurrences of two distinct values in `VARIABLES.var` or `VALUES.val` can be swapped; all occurrences of a value in `VARIABLES.var` or `VALUES.val` can be renamed to any unused value.

Usage

The article [305], which originally introduced the `stretch` constraint, quotes rostering problems as typical examples of use of this constraint.

Remark

We split the original `stretch` constraint into the `stretch_path` and the `stretch_circuit` constraints that respectively use the *PATH LOOP* and the *CIRCUIT LOOP* arc generators. We also reorganise the parameters: the `VALUES` collection describes the attributes of each value that can be assigned to the variables of the `stretch_path` constraint. Finally we skipped the pattern constraint that tells what values can follow a given value. A extension of this constraint (i.e., stretch plus pattern), called `forced_shift_stretch`, where one can specify for each value v with a 0-1 variable, whether it should occur at least once or not at all, was proposed in [209]. By reduction to Hamiltonian path it was shown that enforcing arc-consistency for `forced_shift_stretch` is NP-hard [209].

Algorithm

A first filtering algorithm was described in the original article of G. Pesant [305]. A second filtering algorithm, based on dynamic programming, achieving arc-consistency is depicted in [208, 209]. It also handles the fact that some values can be followed by only a given subset of values. An other alternative achieving arc-consistency is to use the automaton described in the **Automaton** slot.

Systems stretchPath in **Choco**, stretch in **JaCoP**.

See also **common keyword:** `change_continuity`, `group` *(timetabling constraint)*, `group_skip_isolated_item` *(timetabling constraint,sequence)*, `min_size_full_zero_stretch` *(sequence)*, `pattern` *(sliding sequence constraint,timetabling constraint)*, `sliding_distribution` *(sliding sequence constraint)*, `stretch_circuit` *(sliding sequence constraint,timetabling constraint)*.

generalisation: `stretch_path_partition` *(*`variable` *replaced by* `variable` $\in$ `partition`*)*.

uses in its reformulation: `stretch_circuit`.

Keywords **characteristic of a constraint:** automaton, automaton without counters, reified automaton constraint.

combinatorial object: sequence.

constraint network structure: Berge-acyclic constraint network.

constraint type: timetabling constraint, sliding sequence constraint.

filtering: dynamic programming, arc-consistency.

final graph structure: consecutive loops are connected.

For all items of `VALUES`:

Arc input(s) `VARIABLES`

Arc generator $PATH \mapsto$ `collection`(`variables1`, `variables2`)
$LOOP \mapsto$ `collection`(`variables1`, `variables2`)

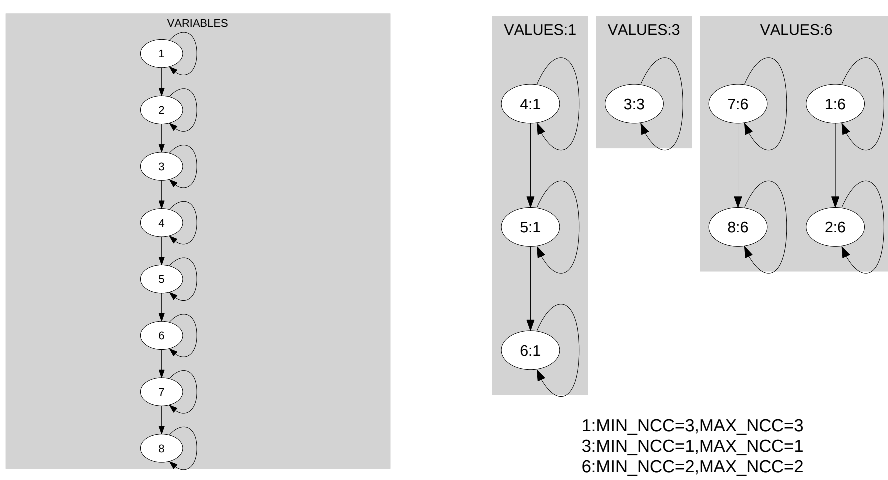
Arc arity 2

Arc constraint(s)
- `variables1.var` = `VALUES.val`
- `variables2.var` = `VALUES.val`

Graph property(ies)
- `not_in`(**MIN_NCC**, 1, `VALUES.lmin` − 1)
- **MAX_NCC** ≤ `VALUES.lmax`

Graph model Part (A) of Figure 5.734 shows the initial graphs associated with values 1, 2, 3 and 6 of the **Example** slot. Part (B) of Figure 5.734 shows the corresponding final graphs associated with values 1, 3 and 6. Since value 2 is not assigned to any variable of the `VARIABLES` collection the final graph associated with value 2 is empty. The `stretch_path` constraint holds since:

- For value 1 we have one connected component for which the number of vertices 3 is greater than or equal to 2 and less than or equal to 4,
- For value 2 we do not have any connected component,
- For value 3 we have one connected component for which the number of vertices 1 is greater than or equal to 1 and less than or equal to 6,
- For value 6 we have two connected components that both contain two vertices: this is greater than or equal to 2 and less than or equal to 2.



(A) (B)

Figure 5.734: Initial and final graph of the `stretch_path` constraint

During the presentation of this constraint at CP'2001 the following point was mentioned: it could be useful to allow domain variables for the minimum and the maximum values of a stretch. This could be achieved in the following way: the `lmin` (respectively `lmax`) attribute would now be a domain variable that gives the size of the shortest (respectively longest) stretch. Finally within the **Graph property(ies)** slot we would replace $\geq$ (and $\leq$) by $=$.

Automaton Let n and m respectively denote the quantities $|\texttt{VARIABLES}|$ and $|\texttt{VALUES}|$. Furthermore, let $\texttt{val}_i$, $\texttt{lmin}_i$ and $\texttt{lmax}_i$, $(i \in [1, m])$, respectively be shortcuts for the expressions $\texttt{VALUES}[i].\texttt{val}$, $\texttt{VALUES}[i].\texttt{lmin}$ and $\texttt{VALUES}[i].\texttt{lmax}$. Without loss of generality, we assume that all the `lmin` attributes of the items of the `VALUES` collection are at least equal to 1. The following automaton $\mathcal{A}$ involving $1 + \texttt{lmax}_1 + \texttt{lmax}_2 + \cdots + \texttt{lmax}_m$ states only accepts solutions to the `stretch_path` constraint. Automaton $\mathcal{A}$ has the following states:

- an initial state s that is also an accepting state,
- $\forall i \in [1, m], \forall j \in [1, \texttt{lmin}_i - 1]$, a non-accepting state $s_{i,j}$,
- $\forall i \in [1, m], \forall j \in [\texttt{lmin}_i, \texttt{lmax}_i]$, an accepting state $s_{i,j}$.

Transitions of $\mathcal{A}$ are defined in the following way:

- $\forall i \in [1, m]$, a transition from s to $s_{i,1}$ labelled by condition $X_l = \texttt{val}_i$,
- a transition from s to s labelled by condition $X_l \neq \texttt{val}_1 \wedge X_l \neq \texttt{val}_2 \wedge \cdots \wedge X_l \neq \texttt{val}_m$,
- $\forall i \in [1, m], \forall j \in [\texttt{lmin}_i, \texttt{lmax}_i]$, a transition from $s_{i,j}$ to s labelled by condition $X_l \neq \texttt{val}_1 \wedge X_l \neq \texttt{val}_2 \wedge \cdots \wedge X_l \neq \texttt{val}_m$,
- $\forall i \in [1, m], \forall j \in [1, \texttt{lmax}_i - 1]$, a transition from $s_{i,j}$ to $s_{i,j+1}$ labelled by condition $X_l = \texttt{val}_i$,
- $\forall i \in [1, m], \forall j \in [\texttt{lmin}_i, \texttt{lmax}_i], \forall k \neq i \in [1, m]$, a transition from $s_{i,j}$ to $s_{k,1}$ labelled by condition $X_l = \texttt{val}_k$.

Figure 5.735 depicts the automaton associated with the `stretch_path` constraint of the **Example** slot. Transitions labels 0, 1, 2, 3 and 4 respectively correspond to the conditions $X_l \neq 1 \wedge X_l \neq 2 \wedge X_l \neq 3 \wedge X_l \neq 6$, $X_l = 1$, $X_l = 2$, $X_l = 3$, $X_l = 6$ (since values 1, 2, 3 and 6 respectively correspond to the values of the first, second, third and fourth item of the `VALUES` collection). The `stretch_path` constraint holds since the corresponding sequence of visited states, s s_{41} s_{42} s_{31} s_{11} s_{12} s_{13} s_{41} s_{42}, ends up in an accepting state (i.e., accepting states are denoted graphically by a double circle in the figure).

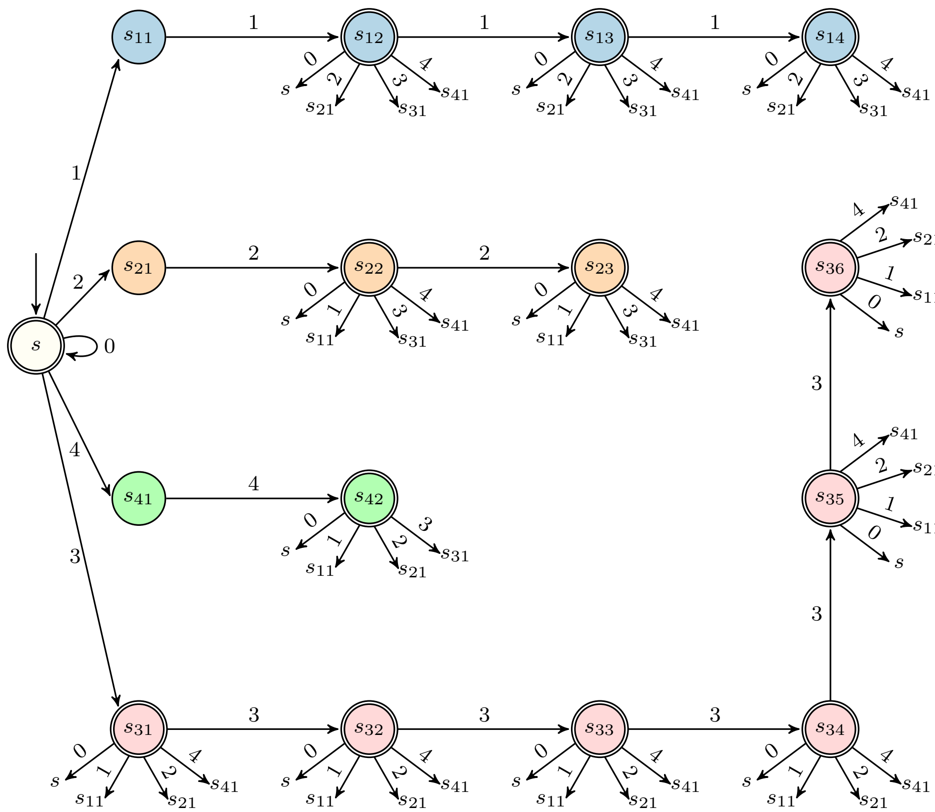


Figure 5.735: Automaton of the `stretch_path` constraint of the **Example** slot (states related to a same stretch have the same colour)